MHT CET

Crash Course Series

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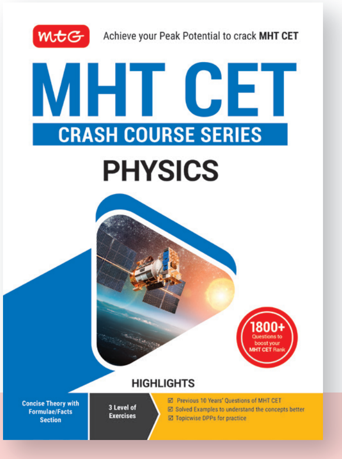
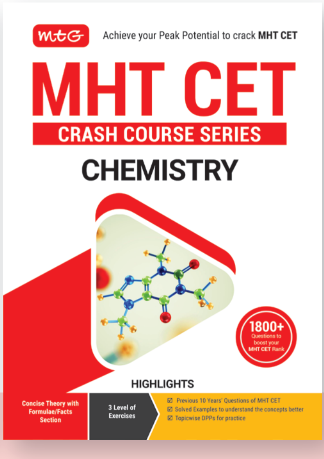
CRASH COURSE SERIES

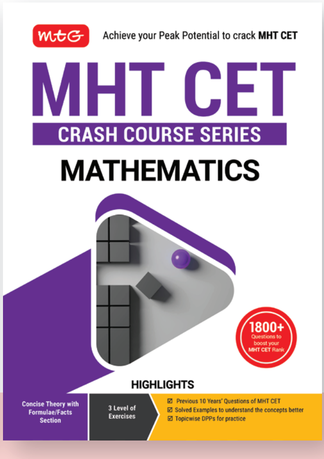
HIGHLIGHTS

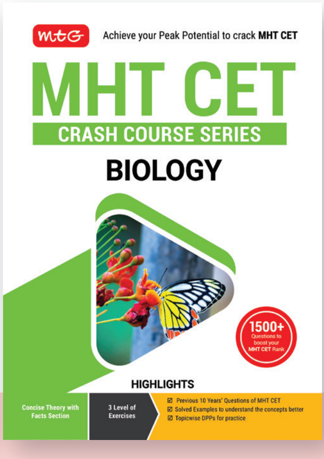
- ☑ Previous 10 Years' Questions of MHT CET
- ☑ Solved Examples to understand the concepts better
- ☑ Topicwise DPPs for practice

7000+ Questions to boost your **MHT CET** Rank









Comprehensive Formulae and Mind Maps

- Effortless Revision: Concise formulae sheets and important notes for quick and effective revision.
- Concept Clarity: Focused on 30% theory to ensure understanding of core concepts without overwhelming details.

Extensive Coverage of Previous Year Questions (PYQs)

- Exam-Focused Preparation: Includes PYQs per chapter, meticulously curated to cover both Class 11 (minimum 10) and Class 12 (minimum 15) questions.
- Insights from Trends: Gain insights into recurring topics and question patterns for a smarter strategy.

Step-by-Step Solved Examples

- Guided Learning: A minimum of 10 solved examples per chapter to break down complex problems and boost problem-solving confidence.
- Enhanced Understanding: Solutions designed to build clarity, precision, and efficiency in answering.

Daily Practice Problems (DPPs)

- Mastery Through Practice: At least 50 topic-wise problems per chapter, handpicked to strengthen your grasp on every topic.
- Instant Solutions: Solutions provided in soft copy format for self-assessment and time-saving review.

Targeted for MHT CET Success

- Tailored for Exam Success: Every component designed specifically for MHT CET, ensuring relevance and high success probability.
- Balanced Approach: Perfect mix of theory, practice, and exam-oriented questions for holistic preparation.

Structured, High-Yield Material

- Time-Efficient: A well-organized course that prioritizes quality over quantity, making it ideal for crash course preparation.
- Boosts Confidence: Regular practice ensures familiarity with exam formats, reducing last-minute stress.

Digital Support for Flexibility

- Soft Copy Solutions: Easy access to answers and explanations for seamless learning anywhere, anytime.
- Interactive Practice: Topic-wise DPPs to monitor progress and identify weak areas quickly.

SAMPLE CHAPTERS

PHYSICS

CHAPTER 1

Motion in a Plane

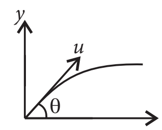
Important Formulae/Facts

- $Speed = \dfrac{Total\ path\ length}{Time\ taken}$
  - $Average\ speed = \dfrac{Total\ distance\ travelled}{Total\ time\ taken}$

    i.e., $v_{av} = \dfrac{s_1 + s_2 + s_3......}{t_1 + t_2 + t_3......}$
  - $Instantaneous\ speed = \lim\limits_{\Delta t \to 0} \dfrac{\Delta s}{\Delta t} = \dfrac{ds}{dt}$
- $Velocity, v = \dfrac{Displacement}{Time\ taken}$
  - $Average\ velocity, (v_{av}) = \dfrac{Total\ displacement}{Total\ time\ taken}$
  - *Slope of displacement-time graph gives velocity of the particle.*
- $Acceleration,\ a = \dfrac{Change\ in\ velocity}{Time\ taken}$
  - $Average\ acceleration,\ a_{av} = \dfrac{\Delta v}{\Delta t}$
  - *Instantaneous acceleration,*

    $a = \lim\limits_{\Delta t \to 0}\left(\dfrac{\Delta v}{\Delta t}\right) = \dfrac{dv}{dt}$
  - *Area under velocity-time graph on the time axis gives displacement of particle.*
- *Equation of motion for a uniform accelerated motion*
  - $v = u + at$
  - $s = ut + \dfrac{1}{2}at^2$
  - $v^2 - u^2 = 2as$
  - $s_n = u + \dfrac{a}{2}(2n - 1)$
- *Relative velocity*
  - *If two bodies are moving along the same line in the same direction with velocities* v_A *and* v_B *relative to earth, the velocity of B relative to A will be given by,* $v_{BA} = v_B - v_A$.
  - *If the bodies are moving towards or away from each other, i.e., directions of* v_A *and* v_B *are opposite, velocity of B relative to A will be given by*

    $v_{BA} = v_B - (-v_A) = v_B + v_A$.
- *A projectile projected with velocity u at an angle* θ *with the horizontal*



  - $x = u\cos\theta t\ ;\ \ y = u\sin\theta t - \dfrac{1}{2}gt^2$
  - *Equation of trajectory,* $y = x\tan\theta - \dfrac{gx^2}{2u^2\cos^2\theta}$
  - *Velocity of the projectile at any time t,*

    $v = \sqrt{(u\cos\theta)^2 + (u\sin\theta - gt)^2}$

    $= \sqrt{u^2 + g^2t^2 - 2gtu\sin\theta}$

    This velocity make angle β *with horizontal.* $\tan\beta = \dfrac{u\sin\theta - gt}{u\cos\theta}$
  - *Horizontal range,* $R = \dfrac{u^2\sin 2\theta}{g}$
  - *For maximum horizontal range,* $\theta = 45°$, $R_m = \dfrac{u^2}{g}$
  - *Time of ascent = time of descent* $= \dfrac{u\sin\theta}{g}$
  - *Time of flight,* $T = \dfrac{2u\sin\theta}{g}$
  - *Maximum height,* $H = \dfrac{u^2\sin^2\theta}{2g}$
- *When the horizontal range is maximum, the time of flight,* $T = \dfrac{2u\sin 45°}{g} = \dfrac{\sqrt{2}u}{g}$
- *When the horizontal range is maximum, the maximum height,*

  $H = \dfrac{u^2\sin^2 45°}{2g} = \dfrac{1}{4}\dfrac{u^2}{g} = \dfrac{R_m}{4}$
- *Uniform Circular Motion*
  - *A particle moves along a circular path with a constant speed.*
  - *Angular displacement,* $\theta = \dfrac{s}{r}$
  - *Angular velocity,* $\omega = \dfrac{\theta}{t}$

Also, $\omega = \frac{2\pi}{T} = 2\pi\upsilon$

- *Linear velocity,* $v = r\omega$
- *Centripetal acceleration,* $a = \frac{v^2}{r} = r\omega^2$
- *Linear acceleration,* $a = r\alpha$

- *Conical Pendulum*
  - *Angular speed* $(\omega) = \sqrt{\frac{g}{l\cos\theta}}$
  - *Time period* $(T) = \frac{2x}{\omega} = 2x\sqrt{\frac{l\cos\theta}{g}}$,

  where l is length of pendulum.

Important Notes

Introduction

- An object is said to be in motion if it changes its position with time, with respect to its surroundings.
- An object can be considered as a point object during motion in a given time, if it covers a distance much greater than its own size.
- The motion of an object is said to be one dimensional motion if only one out of the three coordinates specifying the position of the object changes with respect to time. In such a motion, an object moves along a straight line.
- The motion of an object is said to be two dimensional motion if two out of the three coordinates specifying the position of the object change with respect to time. In such a motion, the object moves in a plane.
- The motion of an object is said to be three dimensional motion if all the three coordinates specifying the position of the object change with respect to time. In such a motion, the object moves in a space.

Rectilinear Motion

- If a particle is restricted to move along a straight line, its motion is called rectilinear motion.
- The distance travelled by an object is defined as the length of the actual path traversed by an object during motion in a given interval of time. Distance is a scalar quantity. Its value can never be zero or negative, during the motion of an object.
- The displacement of an object in a given interval of time is defined as the change in the position of the object along a particular direction during that time. It is the straight and shortest path between the final and initial positions of a body. The displacement of an object can be positive, zero or negative.
- Rest and motion are relative terms. It means an object in one frame of reference may be at rest, the same object can be in motion in another frame of reference.
- The speed of an object is defined as the time rate of change of position of the object in any direction, *i.e.*, speed = distance travelled/time taken.
  Speed is a scalar quantity. It can be zero or positive but never negative.
- An object is said to be moving with a uniform speed, if it covers equal distances in equal intervals of time, howsoever small these intervals may be.
- An object is said to be moving with a variable speed if it covers equal distances in unequal intervals of time or unequal distances in equal intervals of time, howsoever small these intervals may be.
- The average speed of an object for the given motion is defined as the ratio of the total distance travelled by the object to the total time taken.
  $$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$
- The speed of an object at a given instant of time is called its instantaneous speed, $v = \frac{ds}{dt}$.
- The velocity of an object is defined as the rate of change of displacement of the object.
  Velocity = Displacement/Time taken.
  $$\text{Velocity } (\vec{v}) = \frac{\text{Displacement}}{\text{Time}} = \frac{d\vec{x}}{dt}$$
  $$\text{or, } \vec{v} = \frac{\vec{x}_2 - \vec{x}_1}{t_2 - t_1} \text{ m/s}$$
  The velocity is a vector quantity. The velocity of an object can be positive, zero or negative.
- If an object undergoes equal displacements in equal intervals of time, it is said to be moving with a uniform velocity. If an object undergoes unequal displacements in equal intervals of time or equal displacements in unequal intervals of time, it is said to be moving with a variable velocity.
- The average velocity of an object is equal to the ratio of the total displacement, to the total time taken for which the motion takes place.
  $$\text{Average velocity} = \frac{\text{Total displacement}}{\text{Total time taken}}$$

- The velocity of an object at a given instant of time is called instantaneous velocity. When a body is moving with a uniform velocity, its instantaneous velocity is same as that of average uniform velocity.
- The acceleration of an object is defined as the time rate of change of velocity of the object, *i.e.*,
  Acceleration = Change in velocity/Time taken.
  Acceleration is a vector quantity. Acceleration is positive, if the velocity is increasing and is negative if velocity is decreasing. The negative acceleration is called retardation or deceleration.

$$\vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$$

- If the velocity of an object changes by equal amounts in equal intervals of time, it is said to be moving with a uniform acceleration. If the velocity of an object changes by unequal amounts in equal intervals of time, it is said to be moving with a variable acceleration.
- The average acceleration of an object for a given motion is defined as the ratio of the total change in velocity of the object during motion to the total time taken.

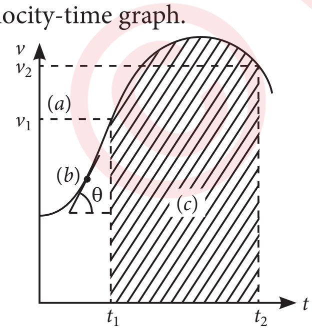
$$\text{Average acceleration} = \frac{\text{Total change in velocity}}{\text{Total time taken}}$$

- The acceleration of an object at a given instant or at a given point of motion is called its instantaneous acceleration. It is defined as the first time derivative of velocity at a given instant or it is also equal to the second time derivative of the position of the object at a given instant.

  Instantaneous acceleration, $\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{d^2 \vec{x}}{dt^2}$

Equations of Motion and their Graphs

Consider a velocity-time graph.



The velocity-time graph gives three pieces of information.

(a) The instantaneous velocity *e.g.*, at t_1, the instantaneous velocity is v_1 and at t_2, it is v_2.

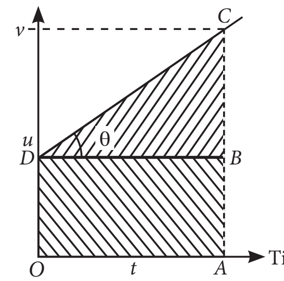
(b) The slope of the tangent to the curve at any point gives the instantaneous acceleration at that point.

$$a = \frac{dv}{dt} = \tan\theta$$

(c) The shaded area under the curve gives total displacement of the object, $s = \int_{t_1}^{t_2} v\,dt$.

Uniformly Accelerated Motion

Let us consider the velocity-time graph of uniform acceleration.



The acceleration (a) is the slope of graph here.

$a = \tan\theta$

$$= \frac{BC}{BD} = \left(\frac{v-u}{t}\right)$$

$\Rightarrow \quad v = u + at \qquad$ ...(i)

The total displacement of the object is the area *OABCDO*.

s = Area *OABCDO* = Area *OABDO* + Area *BCDB*

$$= ut + \frac{1}{2}(v-u)\times t = ut + \frac{1}{2}(at)\times t$$

$\Rightarrow \quad s = ut + \frac{1}{2}at^2 \qquad$ ...(ii)

Again, s = Area *OABCDO* (a trapezium)

$= \frac{1}{2}\times$(Sum of the parallel sides) $\times$
(Perpendicular distance between them)

$$= \frac{1}{2}(AC + OD)\times OA = \frac{1}{2}(v+u)\times t = \frac{1}{2}(v+u)\times\frac{(v-u)}{a}$$

$s = \frac{(v^2-u^2)}{2a}$ or $v^2 = u^2 + 2as \qquad$ ...(iii)

Average velocity, $<v>$

$$<v> = \frac{s}{t} = \frac{ut + \frac{1}{2}at^2}{t} = u + \frac{at}{2} = \frac{2u+at}{2} = \frac{u+(u+at)}{2}$$

$$<v> = \frac{u+v}{2}$$

For displacement in n^{th} second, we subtract the displacement in $(n-1)$ seconds from the displacement in n seconds.

$$s_n = un + \frac{1}{2}an^2$$

$$s_{n-1} = u(n-1) + \frac{1}{2}a(n-1)^2$$

$$s_{n^{th}} = s_n - s_{n-1} = \left[un + \frac{1}{2}an^2\right] - \left[u(n-1) + \frac{1}{2}a(n-1)^2\right]$$

$$= [un - u(n-1)] + \frac{1}{2}a[n^2 - (n-1)^2]$$

$$= u + \frac{1}{2}a[(n+n-1)]$$

$$\Rightarrow \quad s_{n^{th}} = u + \frac{a}{2}(2n-1)$$

Relative Velocity in one Dimension

Let two objects A and B moving uniformly with average velocities v_A and v_B in one dimension, say along x-axis. The velocities mentioned here are assumed to be measured with respect to the ground.

$$x_A = x_A(0) + v_A t,\ x_B = x_B(0) + v_B t$$

$$x_{BA}(t) = [x_B(0) - x_A(0)] + (v_B - v_A)t$$

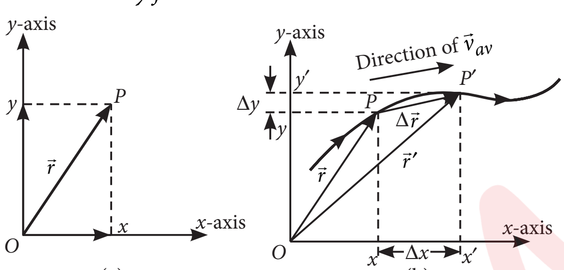
This equation tells us that as seen from object A, object B has a velocity $(v_B - v_A)$ since the displacement from A to B changes steadily by the amount $(v_B - v_A)$ in each unit of time. We say that velocity of object B with respect to object A is $v_B - v_A, \vec{v}_{BA} = \vec{v}_B - \vec{v}_A$

Motion in Two Dimensions-Motion in a Plane

Here, we will learn about describing motion in two dimensions using vectors.

- **Position vector :** The position vector of a particle P located in x-y plane with respect to origin O is as shown in figure (a)

  $$\vec{r} = x\hat{i} + y\hat{j}$$



(a) (b)

- **Displacement vector :** Suppose particle moves from point P to P' along the curve. Then, the displacement is

  $$\Delta\vec{r} = \vec{r'} - \vec{r}$$

  It is directed from P to P'.

  In a component form,

  $$\Delta\vec{r} = (x'\hat{i} + y'\hat{j}) - (x\hat{i} + y\hat{j}) = \hat{i}\Delta x + \hat{j}\Delta y$$

- **Velocity :** The average velocity $(\vec{v}_{av})$ of a particle is ratio of the displacement and corresponding time interval :

  $$\vec{v}_{av} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\Delta x\hat{i} + \Delta y\hat{j}}{\Delta t} = \hat{i}\frac{\Delta x}{\Delta t} + \hat{j}\frac{\Delta y}{\Delta t};$$

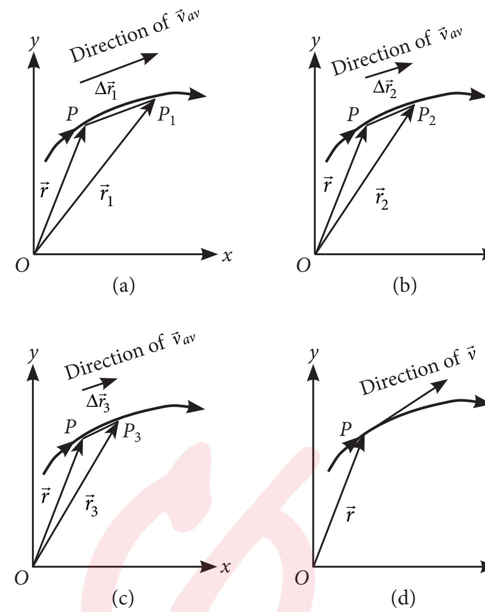
  $$\vec{v}_{av} = \vec{v}_{x_{av}}\hat{i} + \vec{v}_{y_{av}}\hat{j}$$

  The direction of average velocity is the same as that of $\Delta\vec{r}$.

  The velocity (instantaneous velocity) is given by the limiting value of the average velocity as time interval approaches zero.

  $$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

  This limiting process is shown in the figures. As the time interval Δt approaches zero, the average velocity approaches the velocity $\vec{v}$. The direction of $\vec{v}_{av}$ is parallel to the line tangent to the path.



P_1, P_2 and P_3 represent the positions of objects after time Δt_1, Δt_2 and Δt_3. $\Delta\vec{r}_1, \Delta\vec{r}_2$ and $\Delta\vec{r}_3$ are the displacement of the particle in time Δt_1, Δt_2 and Δt_3, respectively. As $\Delta t \to 0$, $\Delta\vec{r}_1 \to 0$ and is along the tangent to the path.

The direction of velocity at any point on the path of a particle is tangential to the path at that point and is in the direction of motion.

$\vec{v}$ in the component form,

$$\vec{v} = \frac{d\vec{r}}{dt} = \lim_{\Delta t \to 0}\left(\frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j}\right)$$

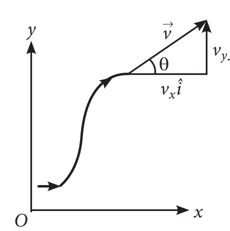
$$\vec{v} = \hat{i}\frac{dx}{dt} + \hat{j}\frac{dy}{dt} = v_x\hat{i} + v_y\hat{j}$$

Magnitude of $\vec{v} = v = \sqrt{v_x^2 + v_y^2}$ and direction of $\vec{v}$,

$$\tan\theta = \frac{v_y}{v_x};\ \theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$

Acceleration

- The average acceleration of a particle for a time interval (Δt) moving in x-y plane is the change in velocity divided by the time interval. The components v_x and v_y of velocity $\vec{v}$ and the angle θ it makes with x-axis is shown in the figure. Then

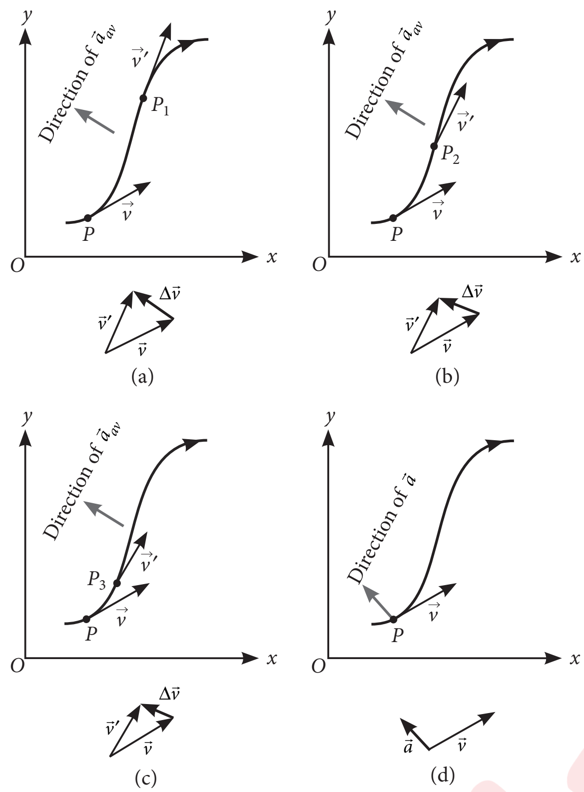


  $$\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\Delta v_x}{\Delta t}\hat{i} + \frac{\Delta v_y}{\Delta t}\hat{j}$$

  $$\vec{a}_{av} = a_{x_{av}}\hat{i} + a_{y_{av}}\hat{j}$$

- The average acceleration for three time intervals (a) Δt_1, (b) Δt_2, and (c) Δt_3, $(\Delta t_1 > \Delta t_2 > \Delta t_3)$ are shown in the figure. In the limiting case as $\Delta t \to 0$, $\Delta v \to 0$

and average acceleration becomes instantaneous acceleration is as shown in figure (d).

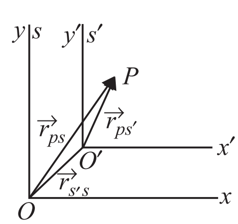


Relative Motion in Two-Dimensions

For a particle in motion with respect to an object, which is also moving with respect to the ground, the velocity of particle with respect to the ground is its actual velocity $(\vec{v}_{\text{act}})$ and the velocity of the particle with respect to the moving object is its relative velocity $(\vec{v}_{\text{rel}})$ and the velocity of moving object (with respect to the ground) is the reference velocity $(\vec{v}_{\text{ref}})$. In the figure, let s and s' be two reference frames with observers at O and O', respectively position of particle P relative to frame s is $\vec{r}_{ps}$ while position of frame s' relative to frame s is $\vec{r}_{s's}$ at a moment.

According to vector law of addition,

$$\vec{r}_{ps} = \vec{r}_{ps'} + \vec{r}_{s's}$$



Differentiating the equation with respect to time, we get

$$\frac{d\vec{r}_{ps}}{dt} = \frac{d\vec{r}_{ps'}}{dt} + \frac{d\vec{r}_{s's}}{dt}$$

so that, $\vec{v}_{ps} = \vec{v}_{ps'} + \vec{v}_{s's}$

Differentiating again, we get

$$\vec{a}_{ps} = \vec{a}_{ps'} + \vec{a}_{s's}$$

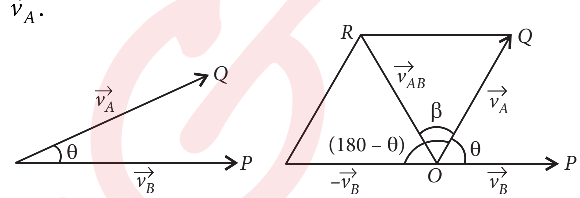
- Relative velocity of a body A with respect to body B, when they are moving in the same direction is given by $\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$
- Relative velocity of a body A with respect to body B when they are moving in the opposite direction is given by $\vec{v}_{AB} = \vec{v}_A + \vec{v}_B$.
- The magnitude of relative velocity of a body A with respect to body B, when they are inclined at an angle θ is given by

$$v_{AB} = \sqrt{(v_A)^2 + (v_B)^2 + 2(v_A)(v_B)\cos(180° - \theta)}$$

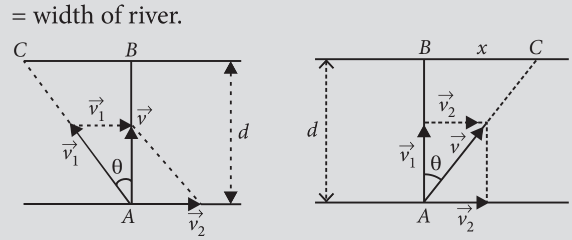
$$= \sqrt{v_A^2 + v_B^2 - 2v_A v_B \cos\theta}$$

$$\tan\beta = \frac{(v_B)\sin(180° - \theta)}{(v_A) + (v_B)\cos(180° - \theta)} = \frac{v_B \sin\theta}{v_A - v_B \cos\theta}.$$

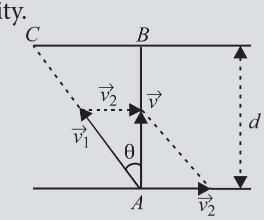
where $\vec{v}_A$ and $\vec{v}_B$ are velocities of two bodies A and B, β is an angle which $\vec{v}_{AB}$ makes with the direction of $\vec{v}_A$.



- **Rain man problem :** If rain is falling vertically with a velocity $\vec{v}_r$ and a man is moving horizontally with speed $\vec{v}_m$, the man can protect himself from the rain if he holds his umbrella in the direction of relative velocity of rain w.r.t. man. If θ is the angle which the direction of relative velocity of rain w.r.t. man makes with the vertical, then $\tan\theta = \dfrac{v_m}{v_r}$.
- **River boat problem :** Let $\vec{v}_1$ = velocity of boat in still water, $\vec{v}_2$ = velocity of flow of water in river, d = width of river.



(a) To cross the river in the shortest path : Here it is required that the boat starting from A must reach the opposite point B along the shortest path AB. For the shortest path, the boat should be rowed upstream making an angle θ with AB such that AB gives the direction of resultant velocity.



So, $\sin\theta = \dfrac{v_2}{v_1}$ and $v^2 = v_1^2 - v_2^2$

Also, $t = \frac{d}{v} = \frac{d}{\sqrt{v_1^2 - v_2^2}}$

(b) To cross the river in the shortest time : For the boat to cross the river in the shortest time, the boat should be directed along AB. Let v be the resultant velocity making an angle θ with AB.

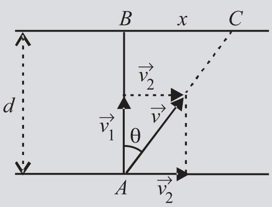
Then, $\tan\theta = \frac{v_2}{v_1}$ and $v^2 = v_1^2 + v_2^2$

Time of crossing, $t = d/v_1$.

Now the boat reaches the point C rather than reaching point B.

If $BC = x$, then

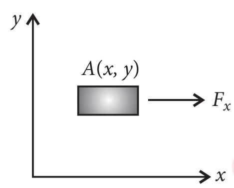
$\tan\theta = \frac{v_2}{v_1} = \frac{x}{d}$ or $x = d \times \left(\frac{v_2}{v_1}\right)$



Projectile Motion

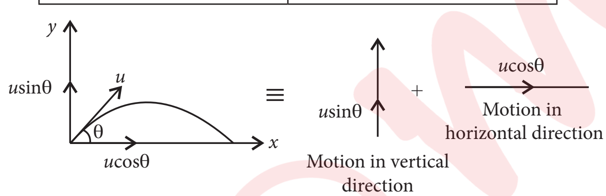
As we know, two dimensional motion can be resolved in two linear motions in two mutually perpendicular directions.

Characteristically, motion in one direction is independent of motion in other mutually perpendicular direction.



A Projectile Projected with Velocity *u* at an angle θ.

x-direction	*y*-direction
$u_x = u\cos\theta;\ a_x = 0$	$u_y = u\sin\theta;\ a_y = -g$



Velocities : Consider an object, thrown or projected with initial velocity u making an angle θ with the horizontal.

Along x-axis, initial velocity, $u_x = u\cos\theta$ (throughout constant)

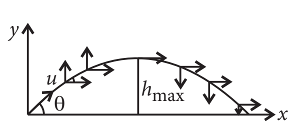
Along y-axis, initial velocity, $u_y = u\sin\theta$

Accelerations :

Along x-axis, $a_x = 0$

$\therefore u_x = v_x = u\cos\theta$ (constant velocity)

Along y-axis, $a_y = -g$



Equation of trajectory : From kinematic equation along x-axis,

$x = u_x t + \frac{1}{2}a_x t^2;\ x = u\cos\theta t \quad (\because a_x = 0)$

$$\Rightarrow t = \frac{x}{u\cos\theta} \quad \text{...(i)}$$

Now along y-axis,

$y = u_y t + \frac{1}{2}a_y t^2;\ y = u\sin\theta t + \frac{1}{2}(-g)t^2$

$$\Rightarrow y = (\tan\theta)x - \frac{g}{(u^2\cos^2\theta)}x^2 \Rightarrow y = ax - bx^2 \quad \text{...(ii)}$$

Here, $a = \tan\theta$ and $b = \frac{g}{u^2\cos^2\theta}$, both are constants.

Equation (ii) is parabolic in nature. Hence, path taken or trajectory is parabolic for a projectile.

Time of maximum height (t_m) : The time at which the projectile reaches the maximum height.

At h_{max}, final velocity along y-axis becomes zero, *i.e.*, $v_y = 0$.

$\therefore v_y = u_y + a_y t_m;\ 0 = u\sin\theta - gt_m \Rightarrow t_m = \frac{u\sin\theta}{g}$

Time of flight (T_f) : Total time in which the projectile is in flight before touching the ground.

Here, $y = 0$

From $y = u_y t + \frac{1}{2}a_y t^2;\ 0 = u\sin\theta T_f - \frac{1}{2}gT_f^2$

$\Rightarrow T_f = \frac{2u\sin\theta}{g}$

Maximum height (h_{max}) : It is the maximum height attained by the projectile during the journey.

At h_{max}, $y = h_{max}$ and $t = t_m$ $\therefore y = u_y t + \frac{1}{2}a_y t^2$

$\Rightarrow h_{max} = u\sin\theta t_m - \frac{1}{2}g t_m^2$

$$h_{max} = \frac{u^2\sin^2\theta}{g} - \frac{1}{2}g\frac{u^2\sin^2\theta}{g^2} \quad \left(\because t_m = \frac{u\sin\theta}{g}\right)$$

$$h_{max} = \frac{u^2\sin^2\theta}{2g}$$

Horizontal range (R) : It is the horizontal distance covered by the projectile before landing.

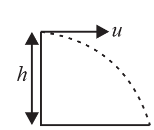
Since, $v_x = \frac{x}{t}$ and along x-axis, $x = R$, $t = T_f$

$\therefore R = v_x \times T_f = u\cos\theta \times \frac{2u\sin\theta}{g} \Rightarrow R = \frac{u^2\sin 2\theta}{g}$

Effects of air resistance : The air resistance decreases the maximum height attained and range of the projectile. It also decreases the speed with which the projectile strikes the ground.

Effect of variation of *g* : Acceleration due to gravity does not remain constant when the range exceeds say 1500 km or so. Then, the direction of g changes because g always points towards the centre of earth. Due to this, shape of trajectory changes from parabolic to elliptical.

A Projectile Projected Horizontally from a Height *h* with Velocity *u*



(i) Time taken by the projectile to reach the ground is $\sqrt{2h/g}$.

(ii) Time taken by the projectile to reach the ground does not depend upon the velocity of projection *i.e.*, u.

(iii) Horizontal range, $x = ut = u\sqrt{\frac{2h}{g}}$.

(iv) Equation of trajectory is $y = \frac{g}{2u^2}x^2$.

(v) Resultant velocity of the projectile at any time t is $v = \sqrt{u^2 + g^2t^2}$.

(vi) Angle made by the resultant velocity with the horizontal is $\tan\beta = \frac{gt}{u}$.

(vii) Velocity of the projectile on striking the ground is $v = \sqrt{u^2 + 2gh}$.

Uniform Circular Motion

When an object is moving on a circular path with a constant speed then motion of the object is said to be uniform circular motion.

Time Period

In circular motion, the time period is defined as the time taken by the object to complete one revolution on its circular path. It is generally denoted by symbol T and its unit is second.

$$T = \frac{\text{Circumference of circle}}{\text{Linear velocity}} = \frac{2\pi r}{v} = \frac{2\pi}{\omega} \quad (\because\ v = r\omega)$$

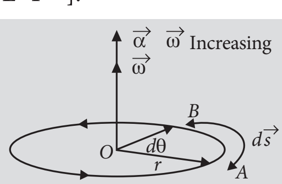
Frequency

- In circular motion, the frequency is defined as the number of revolutions completed by the object on its circular path in a unit time. It is generally denoted by n. Its unit is s^{-1} or hertz (Hz).
- Relation between time period and frequency is $n = \frac{1}{T}$.
- Relation between angular velocity, frequency and time period, $\omega = \frac{2\pi}{T} = 2\pi n$

Angular Velocity

- It is defined as the time rate of change of angular displacement $(\vec{\theta})$ and is given by $\vec{\omega} = \frac{d\vec{\theta}}{dt}$
- It is directed along the axis of rotation. Angular velocity is a vector quantity. Its SI unit is rad s^{-1} and its dimensional formula is $[M^0L^0T^{-1}]$.

The direction of angular displacement and angular velocity is the same for the given particle performing circular motion.



Acceleration

The uniform circular motion (U.C.M.) is called accelerated motion, since the direction of velocity changes continuously but its magnitude remains constant.

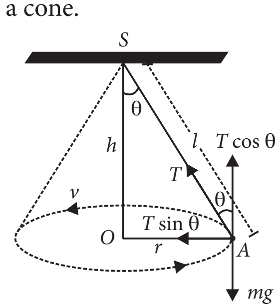
- Acceleration of an object in circular motion has two components :
  - **Tangential acceleration :** The tangential acceleration arises from the change in the speed of an object and has a magnitude given by
    $$a_t = \frac{d|\vec{v}|}{dt}$$
  - **Radial acceleration :** The radial acceleration is due to the change in direction of the velocity and is given by
    $$a_r = \frac{v^2}{r} = \omega^2 r$$
  - **Non uniform circular motion :** For circular motion with variable speed resultant acceleration of the particle is,
    $$a = \sqrt{a_t^2 + a_r^2}$$
    $a_r = \frac{v^2}{r}$, a_t = tangential acceleration

Centripetal acceleration

- Acceleration acting on an object undergoing uniform circular motion is known as centripetal acceleration.
- Centripetal acceleration, $a_c = \frac{v^2}{r} = \omega^2 r$, where r is the radius of the circle.
- This equation can be expressed in vector form as,
  $$\vec{a}_c = -\frac{v^2}{r}\hat{r} = -\omega^2\hat{r}$$
  where $\hat{r}$ is unit vector in a direction of radius vector.
  The negative sign in the expression for acceleration indicates that acceleration and radius vector are oppositely directed.
  It always acts on an object along the radius towards the centre of the circular path.
  Centripetal acceleration is not a constant vector.

Conical Pendulum

Conical pendulum is a simple pendulum which is given such a motion that bob describes a horizontal circle and the string describes a cone.



Consider a heavy metallic sphere of mass m suspended from a light weight string of length l. The metallic sphere is whirled round in a horizontal circle of radius r with a constant speed v. As the body moves along its circular path, the string sweeps over the surface of the cone.

When the metallic bob is at extreme position A, it is acted upon by two forces,

Weight mg of the bob act vertically downward. Tension T in the string is along the length of the string towards the point of suspension.

From figure,

$$T\cos\theta = mg \qquad ...(i)$$

$$T\sin\theta = \frac{mv^2}{r} \qquad ...(ii)$$

Divide (ii) by (i), we get,

$$\frac{T\sin\theta}{T\cos\theta} = \frac{mv^2/r}{mg}$$

$$\therefore \quad v = \sqrt{rg\tan\theta} \qquad ...(iii)$$

Time period of conical pendulum is given by, $T_1 = \frac{2\pi}{\omega}$

$$\text{or} \quad T_1 = \frac{2\pi}{v/r} = \frac{2\pi r}{v} \qquad \left(\because \omega = \frac{v}{r}\right)$$

$$\text{or} \quad T_1 = \frac{2\pi r}{\sqrt{rg\tan\theta}} = 2\pi\sqrt{\frac{r}{g\tan\theta}} \qquad \text{(Using (iii))}$$

From figure, $\sin\theta = \frac{r}{l}$, $\therefore$ $r = l\sin\theta$.

$$\text{or} \quad T_1 = 2\pi\sqrt{\frac{l\sin\theta}{\frac{g\sin\theta}{\cos\theta}}} \quad \therefore \quad T_1 = 2\pi\sqrt{\frac{l\cos\theta}{g}}$$

From the geometry of figure, $h = l\cos\theta$

$$\therefore \quad T_1 = 2\pi\sqrt{\frac{h}{g}} \qquad ...(vi)$$

Thus, time period of conical pendulum is the same as that of simple pendulum of length h, where h is the axial height of cone.

The time period of conical pendulum depends upon angle of inclination to vertical (as angle of inclination increases, period decreases), acceleration due to gravity and length of the pendulum. It does not depend on mass m of bob.

If θ is very very small, then $\cos\theta$ is nearly equal to 1.

Then, $T_1 = 2\pi\sqrt{\frac{l}{g}}$

Squaring and adding (i) and (ii) we get,

$$T^2\cos^2\theta + T^2\sin^2\theta = (mg)^2 + \left(\frac{mv^2}{r}\right)^2$$

$$T^2(\sin^2\theta + \cos^2\theta) = (mg)^2 + \left(\frac{mv^2}{r}\right)^2$$

$$T^2 = (mg)^2 + \left(\frac{mv^2}{r}\right)^2 \qquad ...(iv)$$

From figure, $\tan\theta = \frac{r}{h}$ ...(v)

From equation (iii) and (v)

$$\frac{v^2}{rg} = \frac{r}{h} \Rightarrow v^2 = \frac{r^2 g}{h}$$

substituting v in equation (iv), we get

$$T^2 = (mg)^2 + \left[\frac{m}{r}\left(\frac{r^2 g}{h}\right)\right]^2$$

$$T^2 = (mg)^2\left[1 + \left(\frac{r}{h}\right)^2\right] = mg\sqrt{1 + \left(\frac{r}{h}\right)^2}$$

Previous Years' Questions

1. The difference between angular speed of minute hand and second hand of a clock is

(a) $\frac{59\pi}{900}$ rad/s (b) $\frac{59\pi}{1800}$ rad/s

(c) $\frac{59\pi}{2400}$ rad/s (d) $\frac{59\pi}{3600}$ rad/s *(2014)*

2. A toy cart is tied to the end of an unstretched string of length 'l'. When revolved, the toy cart moves in horizontal circle with radius '$2l$' and time period T. If it is speeded until it moves in horizontal circle of radius '$3l$' with period T_1, relation between T and T_1 is (Hooke's law is obeyed)

(a) $T_1 = \frac{2}{\sqrt{3}}T$ (b) $T_1 = \sqrt{\frac{3}{2}}T$

(c) $T_1 = \sqrt{\frac{2}{3}}T$ (d) $T_1 = \frac{\sqrt{3}}{2}T$

(2015)

3. A particle of mass 'm' is moving in circular path of constant radius 'r' such that centripetal acceleration is varying with time 't' as K^2rt^2 where K is a constant. The power delivered to the particle by the force acting on it is

(a) $m^2K^2r^2t^2$ (b) mK^2r^2t

(c) mK^2rt^2 (d) mKr^2t *(2015)*

4. A stone tied to the end of a string 100 cm long is whirled in a horizontal circle with a constant speed. If the stone makes 14 revolutions in 25 s, what is the acceleration of the stone?

(a) $\left(\frac{88}{25}\right)^2$ m s^{-2} (b) $\left(\frac{25}{88}\right)^2$ m s^{-2}

(c) $\left(\frac{88}{25}\right)$ m s^{-2} (d) $\left(\frac{25}{88}\right)$ m s^{-2} *(2016)*

5. A projectile is thrown with initial velocity v_0 and angle 30° with the horizontal. If it remains in the air for 1 s, what was it's initial velocity?

(a) 19.6 m/s (b) 9.8 m/s

(c) 4.9 m/s (d) 1 m/s *(2017)*

6. In non-uniform circular motion, the ratio of tangential to radial acceleration is (r = radius of circle, v = speed of the particle, α = angular acceleration)

(a) $\frac{\alpha^2 r^2}{v}$ (b) $\frac{\alpha^2 r}{v^2}$

(c) $\frac{\alpha r^2}{v^2}$ (d) $\frac{v^2}{r^2\alpha}$ *(2018)*

7. In uniform circular motion, when time interval $\delta t \to 0$, the angle between change in velocity $(\delta\vec{v})$ and linear velocity $(\vec{v})$ will be

(a) 0° (b) 90° (c) 180° (d) 45° *(2019)*

8. A particle is performing uniform circular motion along the circumference of a circle of diameter 50 cm with frequency 2 Hz. The acceleration of the particle in m/s^2 is

(a) $2\pi^2$ (b) $8\pi^2$

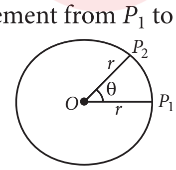
(c) π^2 (d) $4\pi^2$ *(2019)*

9. A particle is moving at a speed of 10 m/s in a circular motion. The radius of the path is 2 m and mass of the particle is 2 kg. The centripetal force of the particle is

(a) 50 N (b) 150 N

(c) 100 N (d) 170 N *(2020)*

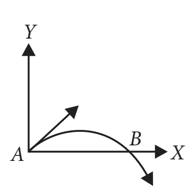
10. A particle is moving on circular path as shown in the figure. Then displacement from P_1 to P_2 is



(a) $2r\cos\frac{\theta}{2}$ (b) $2r\tan\frac{\theta}{2}$

(c) $2\,r\sin\theta$ (d) $2r\sin\frac{\theta}{2}$ *(2020)*

11. The velocity of a projectile at the initial point A is $(2\hat{i}+3\hat{j})$ m/s. It's velocity (in m/s) at point B is



(a) $2\hat{i}-3\hat{j}$

(b) $2\hat{i}+3\hat{j}$

(c) $-2\hat{i}-3\hat{j}$

(d) $-2\hat{i}+3\hat{j}$ *(2021)*

12. A particle is moving on a circular path of 10 m radius. At any instant of time, its speed is 5 m s^{-1} and the speed is increasing at a rate of 2 m s^{-2}. The magnitude of net acceleration at this instant is

(a) 5 m s^{-2} (b) 2 m s^{-2}

(c) 3.2 m s^{-2} (d) 4.3 m s^{-2} *(2021)*

13. A projectile thrown from the ground has initial speed 'u' and its direction makes an angle 'θ' with the horizontal. If at maximum height from ground, the speed of projectile is half its initial speed of projection, then the maximum height reached by the projectile is
[g = acceleration due to gravity, sin 30° = cos 60° = 0.5, cos 30° = sin 60° = $\sqrt{3}/2$]

(a) $\frac{u^2}{g}$ (b) $\frac{u^4}{2g}$ (c) $\frac{2u^2}{g}$ (d) $\frac{3u^2}{8g}$ *(2022)*

14. A particles is performing a uniform circular motion along circle of radius 'R'. In half the period of revolution, its displacement and distance covered are respectively

(a) $\sqrt{2R}, 2\pi R$ (b) $R, \pi R$

(c) $2R, 2\pi R$ (d) $2R, \pi R$ *(2022)*

15. A particle moves around a circular path of radius 'r' with uniform speed 'v'. After moving half the circle, the average acceleration of the particle is

(a) $\frac{v^2}{r}$ (b) $\frac{2v^2}{r}$ (c) $\frac{2v^2}{\pi r}$ (d) $\frac{v^2}{\pi r}$ *(2023)*

16. The position 'x' of a particle varies with a time as $x = at^2 - bt^3$ where 'a' and 'b' are constants. The acceleration of the particle will be zero at time 't' is equal to

(a) $\frac{2a}{3b}$ (b) $\frac{a}{b}$ (c) $\frac{a}{3b}$ (d) zero *(2023)*

17. A particle at rest starts moving with constant angular acceleration 4 rad/s^2 in circular path. At what time the magnitude of its tangential acceleration and centrifugal acceleration will be equal?

(a) 0.4 s (b) 0.5 s

(c) 0.8 s (d) 1.0 s *(2023)*

18. A string of length 'L' fixed at one end carries a body of mass 'm' at the other end. The mass is revolved in a circle in the horizontal plane about a vertical axis passing through the fixed end of the string. The string makes angle 'θ' with the vertical. The angular frequency of the body is 'ω'. The tension in the string is

(a) $mL^2\omega$ (b) $mL\omega^2$ (c) $\frac{\omega^2}{mL}$ (d) $\frac{m\omega^2}{L}$ *(2023)*

19. From the top of a tower of height 40 m, a ball is projected with a speed of 20 m s^{-1} at an angle of elevation 30°. The ratio of the time taken by the ball to come back to the same height and time taken to hit the ground is (Take g = 10 m s^{-2})
(a) 2 : 1 (b) 1 : 2
(c) 4 : 1 (d) 1 : 4 *(2023)*

20. The speed of a swimmer in still water is 20 m/s. The speed of river water is 10 m/s and is flowing due east. If he is standing on the south bank and wishes to cross the river along the shortest path, the angle at which he should make his strokes w.r.t. north is, given by
(a) 45° west (b) 30° west
(c) 0° (d) 60° west *(2023)*

Hints & Explanations

1. (b) : Angular speed, $\omega = \dfrac{\text{Angular distance }(\theta)}{\text{Time taken }(t)}$

For minute hand of a clock, θ = 2π rad, t = 3600 s

$$\therefore \quad \omega_m = \frac{2\pi}{3600} = \frac{\pi}{1800} \text{ rad s}^{-1}$$

For second hand of a clock, θ = 2π rad, t = 60 s

$$\therefore \quad \omega_s = \frac{2\pi}{60} = \frac{\pi}{30} \text{ rad s}^{-1}$$

Required difference between angular speeds

$$= \omega_s - \omega_m$$
$$= \frac{\pi}{30} - \frac{\pi}{1800} = \frac{60\pi - \pi}{1800} = \frac{59\pi}{1800} \text{ rad s}^{-1}$$

2. (d) : The extension in the string in first case

$x_1 = 2l - l = l$

From Hooke's law, $F = kx$

$\therefore \quad m(2l)\omega^2 = kl$

$$\text{or} \quad m(2l)\left(\frac{2\pi}{T}\right)^2 = kl \quad \text{...(i)}$$

In second case, $x_2 = 3l - l = 2l$

$$\therefore \quad m(3l)\left(\frac{2\pi}{T_1}\right)^2 = k(2l) \quad \text{...(ii)}$$

Dividing equation (i) by equation (ii), we get

$$\frac{2}{T_2} \times \frac{T_1^2}{3} = \frac{1}{2}$$

$$\text{or} \quad T_1^2 = \frac{3}{4}T^2 \quad \text{or} \quad T_1 = \frac{\sqrt{3}}{2}T$$

3. (b)

4. (a) : Here, r = 100 cm = 1 m, $\upsilon = \dfrac{14}{25}\text{ s}^{-1}$

$$\therefore \quad \omega = 2\pi\upsilon = 2 \times \frac{22}{7} \times \frac{14}{25} = \frac{88}{25} \text{ rad s}^{-2}$$

Centripetal acceleration,

$$a_c = \omega^2 r = \left(\frac{88}{25}\right)^2 \times 1 = \left(\frac{88}{25}\right)^2 \text{ m s}^{-2}$$

5. (b) : Time of flight, $T = \dfrac{2u\sin\theta}{g}$

Here $u = v_0$, θ = 30°, T = 1 s

$$\therefore \quad T = \frac{2v_0 \sin 30°}{g} \quad \text{or} \quad v_0 = \frac{gT}{2\sin 30°}$$

$$v_0 = \frac{9.8 \times 1}{2 \times \sin 30°} = \frac{9.8 \times 1}{2 \times (1/2)} = 9.8 \text{ m/s}$$

6. (c) : Tangential acceleration , $a_t = \alpha r$

Radial acceleration, $a_r = \dfrac{v^2}{r}$

Now, ratio of tangential to radial acceleration is

$$\frac{a_t}{a_r} = \frac{\alpha r}{v^2/r} = \frac{\alpha r^2}{v^2}$$

7. (b) : In case of uniform circular motion, for time interval $\delta t \to 0$, angle between change in velocity $(\delta\vec{v})$ and linear velocity $(\vec{v})$ will be 90°.

8. (d) : We know that, ω = 2πυ = 4π

$$\therefore \quad \text{Acceleration, } a = \omega^2 r = (4\pi)^2 \times \frac{50 \times 10^{-2}}{2}$$

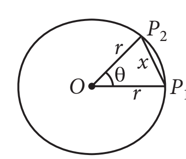
$$= 16\pi^2 \times \frac{50}{200} = 16\pi^2 \times \frac{1}{4} = 4\pi^2 \text{ m/s}^2$$

9. (c) : Given : velocity, v = 10 m/s

Radius, r = 2 m; Mass, m = 2 kg

The centripetal force, $F = \dfrac{mv^2}{r} = \dfrac{2 \times 100}{2} = 100$ N

10. (d) : According to cosine formula



$$\cos\theta = \frac{r^2 + r^2 - x^2}{2r^2}$$

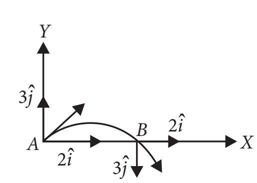
$$\text{or} \quad 2r^2\cos\theta = r^2 + r^2 - x^2$$

$$\text{or} \quad x^2 = 2r^2 - 2r^2\cos\theta = 2r^2\,[1 - \cos\theta]$$

$$= 2r^2\left[2\sin^2\frac{\theta}{2}\right] = 4r^2\sin^2\frac{\theta}{2}$$

Displacement from P_1 to P_2 is $x = 2r\sin\dfrac{\theta}{2}$

11. (a) : At point B, X component of velocity remains unchanged, while Y component reverses it's direction.



∴ The velocity of the projectile at point B is $2\hat{i} - 3\hat{j}$ m/s .

12. (c) : Here, r = 10 m, v = 5 m s^{-1}, a_t = 2 m s^{-2},

$$a_r = \frac{v^2}{r} = \frac{5 \times 5}{10} = 2.5 \text{ m s}^{-2}$$

The net acceleration is

$$a=\sqrt{a_r^2+a_t^2}=\sqrt{(2.5)^2+2^2}=\sqrt{10.25}=3.2\text{ m s}^{-2}$$

13. (d) : At maximum height, $u\cos\theta=\frac{1}{2}u$

$$\Rightarrow\quad \cos\theta=\frac{1}{2}\quad\Rightarrow\quad\theta=60°$$

Maximum height, $H=\frac{u^2\sin^2\theta}{2g}=\frac{u^2\sin^2 60°}{2g}=\frac{3u^2}{8g}$

14. (d) : In half the period, particle is diametrically opposite to its initial position. Hence, its displacement is $2R$.
It has covered a semicircle, hence distance covered by the particle is πR.

15. (c) : Change in velocity $=\vec{v}_2-\vec{v}_1=v-(-v)=2v$

Time, $t=\frac{\pi r}{v}$

Average acceleration $=\frac{2v}{t}=\frac{2vv}{\pi r}=\frac{2v^2}{\pi r}$

16. (c) : Given, $x = at^2 - bt^3$

$$v=\frac{dx}{dt}=2at-3bt^2\ ;\quad A=\frac{dv}{dt}=2a-6bt=0\ ;$$

$$t=\frac{2a}{6b}=\frac{a}{3b}$$

17. (b) : Here : angular acceleration, $\alpha = 4$ rad/s^2

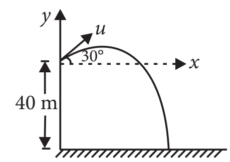
$$\frac{d\omega}{dt}=4\ \Rightarrow\ \omega=4t\text{ rad/s}\ ;\ v=r\omega=4tr$$

Given, $a_T=a_C\ \Rightarrow\ \alpha r=\frac{v^2}{r}\ \Rightarrow\ 4r=\frac{16t^2r^2}{r}$

$$\Rightarrow\quad t^2=\frac{1}{4}\ \Rightarrow\ t=0.5\text{ sec.}$$

18. (b) : Tension, T = centripetal force = $mL\omega^2$

19. (b) : From figure,



The time t_1 is taken by the ball to come back to the same height is

$$t_1=\frac{2u\sin\theta}{g}=\frac{2\times20\times\sin30°}{10}=2\text{ s}$$

Let t_2 be the time taken by the ball to reach to the ground.

For vertical motion, $y=u\sin\theta\,t_2-\frac{1}{2}gt_2^2$

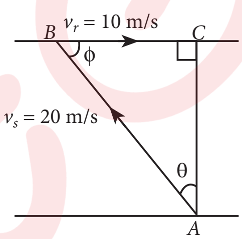
$$\therefore\quad -40=(20\sin30°)\,t_2-\frac{1}{2}\times10\times t_2^2=10t_2-5t_2^2$$

or $\quad t_2^2-2t_2-8=0$

On solving, we get $t_2 = 4$ s

$$\therefore\quad \frac{t_1}{t_2}=\frac{2}{4}=\frac{1}{2}$$

20. (b) :



$$\cos\phi=\frac{10}{20}=\frac{1}{2}\ \text{ or }\ \phi=60°$$

$\Rightarrow\quad \theta = 180° - (90° + 60°) = 30°$
At 30° w.r.t. north swimmer should make his strokes to cross the river.

Solved Examples

Example 1 A body moving in a straight line with uniform acceleration describes three successive equal distances in time intervals t_1, t_2 and t_3, respectively. Show that

$$\frac{1}{t_1}-\frac{1}{t_2}+\frac{1}{t_3}=\frac{3}{t_1+t_2+t_3}$$

Soln.: Let v_1, v_2 and v_3 be the initial velocities of the particle in the time intervals t_1, t_2 and t_3 respectively, and v be the final velocity of the particle in the time interval t_3. The particle moves equal distances in each time interval and let it be d.
Then, average velocity in the time interval t_1 is

$$\frac{d}{t_1}=\frac{1}{2}(v_1+v_2)\qquad\text{...(i)}$$

Similarly, $\frac{d}{t_2}=\frac{1}{2}(v_2+v_3)$ ...(ii)

$$\frac{d}{t_3}=\frac{1}{2}(v_3+v)\qquad\text{...(iii)}$$

and average velocity in the time interval $(t_1 + t_2 + t_3)$ is

$$\frac{3d}{t_1+t_2+t_3}=\frac{1}{2}(v_1+v)\qquad\text{...(iv)}$$

From eqn (i), (ii), (iii) and (iv) we get,

$$\frac{1}{t_1}-\frac{1}{t_2}+\frac{1}{t_3}=\frac{1}{2d}(v_1+v_2)-\frac{1}{2d}(v_2+v_3)+\frac{1}{2d}(v_3+v)$$

$$=\frac{1}{2d}(v_1+v)=\frac{3}{t_1+t_2+t_3}\ \text{(Proved).}$$

[using (iv)]

Example 2 At time t, positions of three particles A, B and C are as follows:

$$x_A = 2t + 7,\ x_B = 3t^2 + 2t + 6,\ x_C = 5t^3 + 4t$$

Which of them has uniform (constant) acceleration?

Soln.: (a) $v = \dfrac{dx_A}{dt} = 2$ and $a = \dfrac{d^2x_A}{dt^2} = 0$

The particle has no acceleration at all.

(b) $v = \dfrac{dx_B}{dt} = (3\times 2t) + 2 + 0 = 6t + 2$; $a = \dfrac{d^2x_B}{dt^2} = 6$

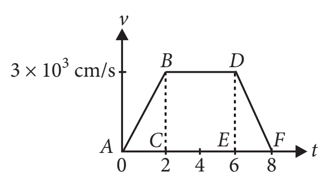
Here, acceleration is uniform.

(c) $v = \dfrac{dx_C}{dt} = 5\times 3t^2 + 4 = 15t^2 + 4$

$$a = \frac{dv}{dt} = \frac{d^2x_C}{dt^2} = 15\times 2t = 30t.$$

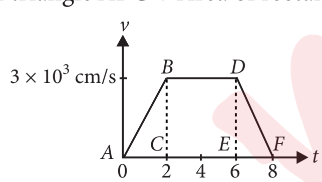
In this case, acceleration depends upon time, so it is not uniform.

Example 3 The velocity-time graph of the motion of a car is given below. Find the distance travelled by the car in the first six seconds. What is the deceleration of the car during the last two seconds?



Soln.: The area enclosed between v-t curve gives the distance travelled by the body.

∴ Distance travelled by the car in the first six seconds
= Area of triangle ABC + Area of rectangle $BDEC$



$$= \frac{1}{2}\times 2\times 3\times 10^3 + (6-2)\times 3\times 10^3$$

$= 3 \times 10^3 + 12 \times 10^3 = 15 \times 10^3$ cm = 150 m.

From straight line equation, $v = u + at$

⇒ $0 = 3 \times 10^3 + a \times 2$

$$a = \frac{-3\times 10^3}{2} = -1.5\times 10^3 \text{ cm/s}^2 = -15 \text{ m/s}^2.$$

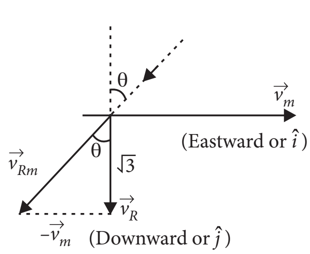
Example 4 A man is moving due to east with a speed 1 km/hr and rain is falling vertically with a speed $\sqrt{3}$ km/hr. At what angle from the vertical the man has to hold his umbrella to keep the rain away? Also find the speed of rain drops w.r.t. man.

Soln.: $\tan\theta = \dfrac{1}{\sqrt{3}}, \theta = 30°$

$\vec{v}_m = (1 \text{ km hr}^{-1})\hat{i}$

$\vec{v}_R = \left(\sqrt{3} \text{ km hr}^{-1}\right)(-\hat{j})$

$\vec{v}_{Rm} = \vec{v}_R - \vec{v}_M = (-\sqrt{3}\,\hat{j} - \hat{i})$

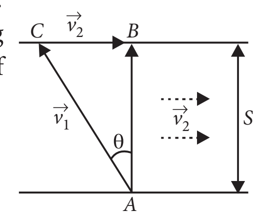


$$|\vec{v}_{Rm}| = \sqrt{(\sqrt{3})^2 + (1)^2} = 2 \text{ km hr}^{-1}$$

Angle of relative velocity with vertical is 30°, so the man has to tilt his umbrella at 30° with vertical towards the east.

Example 5 A river is flowing from west to east at a speed of 5 m/min. In what direction should a man on the south bank of the river, capable of swimming at 10 m/min in still water, should swim to cross the river (i) along the shortest path (ii) in shortest time?

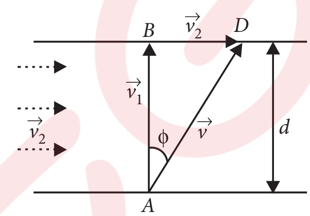
Soln.: (i) Along shortest path (AB):
The man will cross the river along the shortest path if the resultant of $\vec{v}_2$ and $\vec{v}_1$ is along AB



∴ $\sin\theta = \dfrac{v_2}{v_1} = \dfrac{5}{10} = \dfrac{1}{2} = \sin 30°$

∴ θ = 30° west of north.

(ii)



In shortest time:

Time, $t = \dfrac{d}{v_1} = \dfrac{d}{v\cos\phi}$

Time will be minimum if cos ϕ is maximum *i.e.,* ϕ = 0.
The man should swim along AB *i.e.,* perpendicular to the flow.

Example 6 A body of mass m is projected horizontally with a velocity v from the top of a tower of height h and it reaches the ground at a distance x from the foot of the tower. If a second body of mass $2m$ is projected horizontally from the top of a tower of height $2h$, it reaches the ground at a distance $2x$ from the foot of the tower. Then what is the horizontal velocity of the second body?

Soln.: For the first body, $h = \dfrac{1}{2}gt^2$ ...(i)

As, the first body is projected with velocity v
and $x = vt$ ...(ii)

From equations (i) and (ii), we get

$$h = \frac{1}{2}g\left(\frac{x^2}{v^2}\right) \quad ...(iii)$$

For the second body, let v' be the velocity of projection, then

$$2h = \frac{1}{2}g\left[\frac{(2x)^2}{v'^2}\right] \quad ...(iv)$$

Dividing equations (iii) by equation (iv), we get

$$\frac{1}{2} = \frac{x^2}{v^2}\times\frac{v'^2}{4x^2} \text{ or } v' = \sqrt{2}v$$

Example 7 Two bodies are thrown with the same initial speed at angles α and $(90° - \alpha)$ with the horizontal. What will be the ratio of maximum heights attained by them and horizontal ranges?

Soln.: Horizontal range, $R = \frac{u^2}{g}\sin 2\theta$

and maximum height, $H = \frac{u^2 \sin^2\theta}{2g}$

when $\theta = \alpha$; $R_1 = \frac{u^2}{g}\sin 2\alpha$ and $H_1 = \frac{u^2\sin^2\alpha}{2g}$

when $\theta = (90° - \alpha)$;

$$R_2 = \frac{u^2 \sin 2(90° - \alpha)}{g} = \frac{u^2 \sin(180° - 2\alpha)}{g} = \frac{u^2 \sin 2\alpha}{g}$$

and $H_2 = \frac{u^2\sin^2(90° - \alpha)}{2g} = \frac{u^2\cos^2\alpha}{2g}$

$$\therefore \quad \frac{H_1}{H_2} = \frac{\sin^2\alpha}{\cos^2\alpha} = \tan^2\alpha \text{ and } \frac{R_1}{R_2} = 1$$

Example 8 An astronaut is rotating in a rotor of radius 4 m. If he can withstand upto acceleration of 10 g, then what is the maximum number of permissible revolutions? $(g = 10 \text{ m s}^{-2})$

Soln.: In case of uniform circular motion

$$a_r = \frac{v^2}{r} = \omega^2 r \quad [\text{as } v = r\omega]$$

or $a_r = (2\pi\upsilon)^2 r$ [As, $\omega = 2\pi\upsilon$]

or $\upsilon = \frac{1}{2\pi}\sqrt{\frac{a_r}{r}}$

Hence, $a_r < 10$ g, so

$$\upsilon < \frac{1}{2\pi}\sqrt{\frac{10\times 10}{4}} \text{ i.e. } \upsilon_{max} = \left[\frac{5}{2\pi}\right] \text{rev/s}$$

Example 9 A motor car is travelling at 30 m s^{-1} on a circular road of radius 500 m. It is increasing its speed at the rate of 2 m s^{-2}, what is its acceleration?

Soln.: In this problem,

$$a_t = \frac{dv}{dt} = 2 \text{ m s}^{-2}$$

$$a_r = \frac{v^2}{r} = \frac{30\times 30}{500} = 1.8 \text{ m s}^{-2}$$

so, $a = \sqrt{a_r^2 + a_t^2} = \sqrt{(1.8)^2 + (2)^2} = 2.7 \text{ m s}^{-2}$

Example 10 The acceleration of a particle is increasing linearly with time t as bt. The particle starts from the origin with an initial velocity v_0. Find the distance travelled by the particle in time t.

Soln.: Acceleration, $a = \frac{dv}{dt} = bt$; so, $dv = bt dt$ ...(i)

Integrating equation (i) within the conditions of motion we have,

$$\int_{v_0}^{v} dv = \int_0^t bt dt \text{ or } (v - v_0) = \frac{bt^2}{2}$$

or $v = v_0 + \frac{bt^2}{2} = \frac{ds}{dt}$

or $ds = v_0\, dt + \frac{bt^2}{2} dt$ ...(ii)

Integrating equation (ii) we get,

$$s = v_0 t + \frac{1}{6}bt^3$$

Daily Practice Problems

Rectilinear Motion

1. In one dimensional motion, instantaneous speed v satisfies $0 \le v < v_0$.

(a) The displacement in time T must always take non-negative values.

(b) The displacement x in time T satisfies $-v_0 T < x < v_0 T$.

(c) The acceleration is always a non-negative number.

(d) The motion has no turning points.

2. The position of an object moving along x-axis is given by $x = a + bt^2$, where $a = 8.5$ m and $b = 2.5$ m s^{-2} and t is measured in seconds. The instantaneous velocity (in m s^{-1}) of the object at $t = 2$ s is

(a) 5 (b) 10

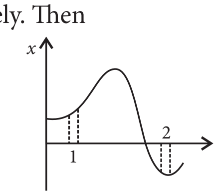
(c) 15 (d) 20

3. The distance travelled by a particle is related to time t as $x = 4t^2$. The velocity of the particle at $t = 5$ s is

(a) 10 m s^{-1} (b) 20 m s^{-1}

(c) 25 m s^{-1} (d) 40 m s^{-1}

4. Figure shows the x-t plot of a particle in one-dimensional motion. Two different equal intervals of time are shown. Let v_1 and v_2 be average speed in time intervals 1 and 2 respectively. Then



(a) $v_1 > v_2$ (b) $v_2 > v_1$

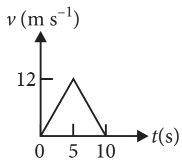
(c) $v_1 = v_2$ (d) data is insufficient

5. A car is going with speed 20 m s^{-1} along north direction. It turns towards east direction in 2 s maintaining the same speed. Find magnitude and direction of average acceleration.

(a) 10 m/s^2 (b) $10\sqrt{2}$ m/s^2

(c) $10\sqrt{3}$ m/s^2 (d) $\frac{10}{\sqrt{2}}$ m/s^2

6. The speed-time graph of a particle moving along a fixed direction is shown in the figure. The distance traversed by the particle between $t = 0$ s to $t = 10$ s is



(a) 20 m (b) 40 m

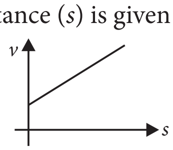
(c) 60 m (d) 80 m

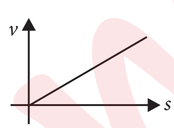
7. A particle is moving along a straight line with constant acceleration. At the end of tenth second its velocity becomes 20 m/s and in tenth second it travels a distance of 10 m. Find the acceleration of the particle.

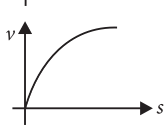
(a) 40 m/s^2 (b) 10 m/s^2

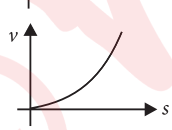
(c) 30 m/s^2 (d) 20 m/s^2

8. A body starting from rest moves along a straight line with a constant acceleration. The variation of speed (v) with distance (s) is given by

(a) 

(b) 

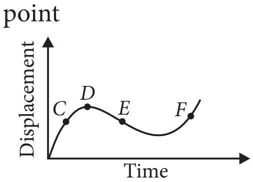
(c) 

(d) 

9. A girl standing on a stationary lift (open from above) throws a ball upwards with initial speed 50 m s^{-1}. The time taken by the ball to return to her hands is (Take $g = 10$ m s^{-2})

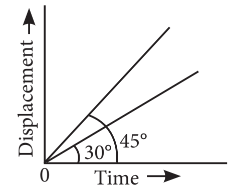
(a) 5 s (b) 10 s (c) 15 s (d) 20 s

10. The displacement-time graph of a moving particle is as shown in figure. The instantaneous velocity of the particle is negative at the point



(a) D (b) F (c) C (d) E

11. The displacement-time graphs of two moving particles make angles of 30° and 45° with the x-axis as shown in the figure. The ratio of their respective velocity is



(a) $\sqrt{3}:1$ (b) 1 : 1

(c) 1 : 2 (d) $1:\sqrt{3}$

Motion in Two Dimensions – Motion in a Plane

12. The x and y co-ordinates of a particle at any time t are given by $x = 7t + 4t^2$ and $y = 5t$ where x and y are in m and t in s. The acceleration of the particle at 5 s is

(a) zero (b) 8 m s^{-2}

(c) 20 m s^{-2} (d) 40 m s^{-2}

13. Four persons K, L, M and N are initially at the corners of a square of side of length d. If every person starts moving with the same speed v such that K is always headed towards L, L towards M, M is headed directly towards N and N towards K, then the four persons will meet after

(a) $\frac{d}{v}$ (b) $\frac{\sqrt{2}d}{v}$

(c) $\frac{d}{\sqrt{2}v}$ (d) $\frac{d}{2v}$

14. A particle motion on a shape curve is governed by $x = 2\sin t$, $y = 3\cos t$ and $z = \sqrt{5}\ \sin t$. What is the magnitude of velocity of the particle at any time t ?

(a) $3\sqrt{2}\sin t$ (b) 3

(c) $3\sqrt{2}\cos t$ (d) $3\sqrt{2}$

15. From the top of a tower of height 40 m, a ball is projected upwards with a speed of 20 m s^{-1} at an angle of elevation of 30°. Then the ratio of the total time taken by the ball to hit the ground to its time of flight is (Take $g = 10$ m s^{-2})

(a) 2 : 1 (b) 3 : 1 (c) 3 : 2 (d) 4 : 1

16. A body of mass m is projected horizontally with a velocity v from the top of a tower of height h and it reaches the ground at a distance x from the foot of the tower. If a second body of mass $3m$ is projected horizontally from the top of a tower of height $3h$, it reaches the ground at a distance $3x$ from the foot of the tower. The horizontal velocity of the second body is

(a) $\sqrt{3}v$ (b) $2v$ (c) $\sqrt{2}v$ (d) $\frac{v}{2}$

17. Two balls are projected making angles of 30° and 45° respectively with the horizontal. If both have same velocity at the highest point of their path, then the ratio of their horizontal ranges is

(a) 1 : 3 (b) 3 : 1

(c) $\sqrt{3}:\sqrt{2}$ (d) $1:\sqrt{3}$

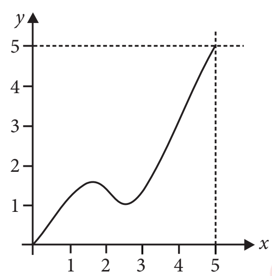
18. The speed of a projectile when it is at its greatest height is $\sqrt{2/5}$ times its speed at half the maximum height. What is its angle of projection?

(a) 30° (b) 60°
(c) 45° (d) 0°

19. A stone is projected with a velocity $20\sqrt{2}$ m s^{-1} at an angle of 45° to the horizontal. The average velocity of stone during its motion from starting point to its maximum height is (g = 10 m s^{-2})

(a) $5\sqrt{5}$ m s^{-1} (b) $10\sqrt{5}$ m s^{-1}
(c) 20 m s^{-1} (d) $20\sqrt{5}$ m s^{-1}

20. A particle moves on a path as shown. The particle takes 10 seconds in going from starting point to the final point. What is the average velocity vector of the particle?



(a) $0.5\hat{i}+\hat{j}$
(b) $0.5\hat{i}+2.5\hat{j}$
(c) $0.5\hat{i}+0.5\hat{j}$
(d) None of these

21. A particle starts from the origin at t = 0 s with a velocity of $10\hat{j}$ m s^{-1} and moves in the $x-y$ plane with a constant acceleration of $(8\hat{i}+2\hat{j})$ m s^{-2}. At an instant when the x-coordinate of the particle is 16 m, y-coordinate of the particle is

(a) 16 m (b) 28 m
(c) 36 m (d) 24 m

22. A body starts from rest with uniform acceleration. The velocity of the body after t seconds is v. The displacement of the body in last three seconds is

(a) $\frac{3v}{2}(t-3)$ (b) $\frac{3v}{2}(t+3)$
(c) $3v\left[1-\frac{3}{2t}\right]$ (d) $3v\left[1+\frac{3}{2t}\right]$

23. An aeroplane flying horizontally at a speed of 98 m s^{-1} releases an object which reaches the ground in 10 s. The angle made by the velocity of the object with the horizontal at the time of hitting the ground is

(a) 30° (b) 45°
(c) 75° (d) 60°

24. A particle is projected from origin in xy-plane at angle 37° with x-axis with a speed of 10 m s^{-1}. Its acceleration is $\vec{a}=\left(-5\hat{i}+10\hat{j}\right)$ m s^{-2}. Find the speed of particle where its x-coordinate is zero.

(a) 10 m s^{-1} (b) 38 m s^{-1}
(c) 6 m s^{-1} (d) 38.8 m s^{-1}

25. Rain is falling vertically with velocity 10 m/s and a man is moving with velocity 6 m/s. Find the angle at which the man should hold his umbrella to avoid getting wet.

(a) $\tan^{-1}\left(\frac{4}{5}\right)$ (b) $\tan^{-1}\left(\frac{5}{3}\right)$
(c) $\tan^{-1}\left(\frac{3}{5}\right)$ (d) $\tan^{-1}\left(\frac{5}{4}\right)$

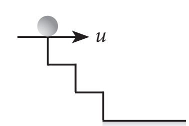
26. A man standing on a road has to hold his umbrella at 30° with the vertical to keep the rain away. He throws the umbrella and starts running at 10 km/h. He finds that raindrops are hitting his head vertically. Find the speed of raindrops with respect to

	the ground	**moving man**
(a)	20 km/h	$10\sqrt{3}$ km/h
(b)	10 km/h	15 k m/h
(c)	$10\sqrt{3}$ km/h	20 km/h
(d)	15 km/h	10 km/h

27. Rain is falling, vertically with a speed of 1 m s^{-1}. Winds starts blowing after sometime with a speed of 1.732 m s^{-1} in east to west direction. In which direction should a boy waiting at a bus stop hold his umbrella?

(a) 30° with the vertical towards the west
(b) 60° with the vertical towards the east
(c) 90° with the vertical towards the east
(d) 45° with the vertical towards the west.

28. A staircase contains three steps each 10 cm high and 20 cm wide as shown in figure. The minimum horizontal velocity of a ball rolling off the uppermost plane so as to hit directly the lowest plane is



(a) 1 m/s
(b) 2 m/s
(c) 3 m/s
(d) 4 m/s

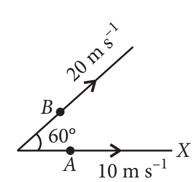
29. The speed of a projectile at its maximum is $\sqrt{3}/2$ times its initial speed. If the range of the projectile is n times the maximum height attained by it, n is equal to

(a) $\frac{4}{3}$ (b) $2\sqrt{3}$ (c) $4\sqrt{3}$ (d) $\frac{3}{4}$

30. A bullet is fired from a gun at the speed of 280 m s^{-1} in the direction 30° above the horizontal. The maximum height attained by the bullet is
(g = 9.8 m s^{-2}, sin 30° = 0.5)

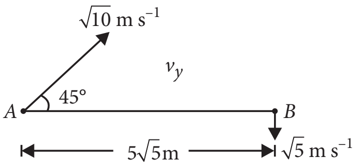
(a) 1000 m (b) 3000 m (c) 2800 m (d) 2000 m

31. Particle A moves along X-axis with a uniform velocity of magnitude 10 m/s. Particle B moves with uniform velocity 20 m/s along a direction making an angle of 60° with the positive direction of X-axis as shown in the figure. The relative velocity of B with respect to that of A is



(a) 10 m/s along X-axis
(b) $10\sqrt{3}$ m/s along Y-axis (perpendicular to X-axis)
(c) $10\sqrt{5}$ m/s along the bisection of the velocities of A and B
(d) 30 m/s along negative X-axis.

32. Two particles starts from A and B as shown in the figure. Then

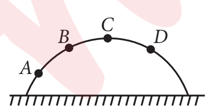


(a) the minimum separation between two particles is achieved in time 1 s
(b) the minimum separation is 15 m
(c) they may collide to each other
(d) they must collide to each other.

33. Three particles A, B and C are projected from the same point with the same initial speeds making angles 30°, 45° and 60° respectively with the horizontal. Which of the following statement is correct ?
(a) A, B and C have unequal ranges
(b) Ranges of A and C are equal and less than that of B
(c) Ranges of A and C are qual and greatest than that of B
(d) A, B and C have equal ranges.

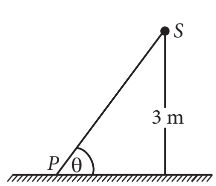
34. A ball is thrown from a point with a speed v_0 at an angle of projection θ. From the same point and at the same instant a person starts running with a constant speed $v_0/2$ to catch the ball. Will the person be able to catch the ball? If yes, what should be the angle of projection?
(a) Yes, 60° (b) Yes, 30°
(c) No (d) Yes, 45°

35. A stone is projected from ground. Its path is as shown in figure. At which point its speed is decreasing at fastest rate ?
(a) A
(b) B
(c) C
(d) D



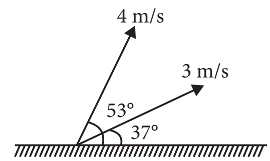
36. Two persons P and Q crosses the river starting from point A on one side to exactly opposite point B on the other bank of the river. The person P crosses the river in the shortest path. The person Q crosses the river in shortest time and walks back to point B. Velocity of river is 3 km/h and speed of each boat is 5 km/h w.r.t. river. If the two persons reach the point B in the same time, then the speed of walk of Q is
(a) 5 km/h (b) 15 km/h
(c) 12 km/h (d) 20 km/h

37. Spotlight S rotates in a horizontal plane with constant angular velocity of 0.1 rad/s. The spot of light P moves along the wall at a distance of 3 m. The velocity of the spot P when θ = 45° (see fig.) is



(a) 1.3 m/s (b) 0.6 m/s
(c) 0.1 m/s (d) 2.2 m/s

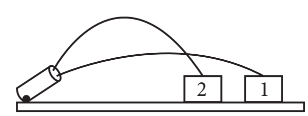
38. Two particles are projected with speed 4 m/s and 3 m/s simultaneously from same point as shown in the figure.



Then
(a) Their relative velocity is along horizontal direction
(b) Their relative acceleration is non-zero and it is along vertical direction
(c) They will hit the surface simultaneously
(d) Their relative velocity is constant and has magnitude 1.4 m/s.

39. The coordinates of a particle moving in a plane are given by $x(t) = a\cos(pt)$ and $y(t) = b\sin(pt)$ where a, b ($< a$) and p are positive constants of appropriate dimensions. Then
(a) the path of the particle is an ellipse
(b) the velocity and acceleration of the particle are normal to each other at $t = \pi/(2p)$
(c) the acceleration of the particle is always directed towards a focus
(d) All of these.

40. Two similar cannon simultaneously fires two identical cannon balls at target 1 and 2 as shown in the figure. If the cannon balls have identical initial speeds, which of the following statement is true?



(a) Hit target 2 before target 1
(b) Hit target 1 before target 2
(c) Both targets hit at the same time
(d) Given information is insufficient

41. A particle of mass m moves along a curve $y = x^2$. When particle has x-coordinate as $\frac{1}{2}$ m and x-component of velocity as 4 m/s then
(a) the position coordinates of particle are $\left(\frac{1}{2}, \frac{1}{7}\right)$
(b) the velocity of particle will be along the line $4x - 4y - 1 = 0$
(c) the magnitude of velocity at that instant is 4 m/s
(d) the magnitude of angular momentum of particle about origin at that position is 0.

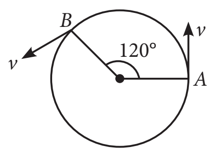
42. A piece of stone is thrown from the top of a tower with a horizontal speed of $10\sqrt{3}$ m/s. It is found that at a point P along the path, the velocity vector of the stone makes an angle 30° with the horizontal. The point P is reached in time t which is given by ($g = 10$ m/s^2)
(a) 1 s (b) $\sqrt{3}$ s
(c) 2 s (d) $2\sqrt{3}$ s

Uniform Circular Motion

43. A particle is moving with constant speed in a circular path. When the particle turns by angle 90°, the ratio of instantaneous velocity to its average velocity is $\pi : x\sqrt{2}$. The value of x will be

(a) 7 (b) 5 (c) 1 (d) 2

44. As shown in the figure, a particle is moving with constant speed π m/s. Considering its motion from A to B, the magnitude of the average velocity is



(a) $\sqrt{3}$ m/s

(b) $2\sqrt{3}$ m/s

(c) $1.5\sqrt{3}$ m/s

(d) π m/s

45. An object moves at a constant speed along a circular path in a horizontal plane with center at the origin. When the object is at $x = +2$ m, its velocity is $-4\hat{j}$ m/s. The object's velocity (v) and acceleration (a) at $x = -2$ m will be

(a) $v = -4\hat{i}$ m/s, $a = -8\hat{j}$ m/s^2

(b) $v = 4\hat{j}$ m/s, $a = 8\hat{i}$ m/s^2

(c) $v = 4\hat{i}$ m/s, $a = 8\hat{j}$ m/s^2

(d) $v = -4\hat{j}$ m/s, $a = 8\hat{i}$ m/s^2

46. A body is moving with constant speed, in a circle of radius 10 m. The body completes one revolution in 4 s. At the end of 3rd second, the displacement of body (in m) from its starting point is

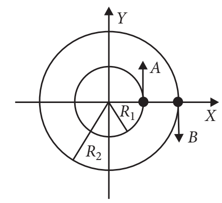
(a) 15π (b) $10\sqrt{2}$ (c) 30 (d) 5π

47. A particle is moving along a circular path with a constant speed of 10 m s^{-1}. What is the magnitude of the change in velocity of the particle, when it moves through an angle of 60° around the centre of the circle?

(a) zero (b) $10\sqrt{2}$ m/s

(c) $10\sqrt{3}$ m/s (d) 10 m/s

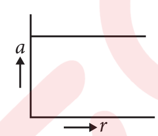
48. Two particles A, B are moving on two concentric circles of radii R_1 and R_2 with equal angular speed ω. At $t = 0$, their positions and direction of motion are shown in the figure. The relative velocity $\vec{v}_A - \vec{v}_B$ at $t = \frac{\pi}{2\omega}$ is given by

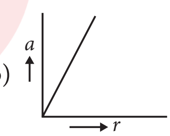


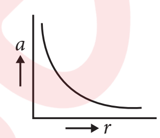
(a) $-\omega(R_1 + R_2)\hat{i}$ (b) $\omega(R_1 - R_2)\hat{i}$

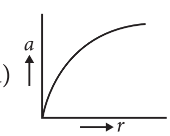
(c) $\omega(R_2 - R_1)\hat{i}$ (d) $-\omega(R_1 + R_2)\hat{i}$

49. If a body moving in a circular path maintains constant speed of 10 m s^{-1}, then which of the following correctly describes relation between acceleration and radius ?

(a) 

(b) 

(c) 

(d) 

50. Two particles A and B are moving in uniform circular motion in concentric circles of radii r_A and r_B with speed v_A and v_B respectively. Their time period of rotation is the same. The ratio of angular speed of A to that of B will be

(a) 1 : 1 (b) $r_A : r_B$

(c) $v_A : v_B$ (d) $r_B : r_A$

ANSWER KEYS

1. (b)	**2.** (b)	**3.** (d)	**4.** (a)	**5.** (b)	**6.** (c)	**7.** (d)	**8.** (c)	**9.** (b)	**10.** (d)
11. (d)	**12.** (b)	**13.** (a)	**14.** (b)	**15.** (a)	**16.** (a)	**17.** (d)	**18.** (b)	**19.** (b)	**20.** (c)
21. (d)	**22.** (c)	**23.** (b)	**24.** (b)	**25.** (c)	**26.** (a)	**27.** (b)	**28.** (b)	**29.** (c)	**30.** (a)
31. (b)	**32.** (a)	**33.** (d)	**34.** (a)	**35.** (a)	**36.** (c)	**37.** (b)	**38.** (d)	**39.** (d)	**40.** (b)
41. (b)	**42.** (a)	**43.** (d)	**44.** (c)	**45.** (b)	**46.** (b)	**47.** (d)	**48.** (c)	**49.** (c)	**50.** (a)

CHEMISTRY

CHAPTER

1

Some Basic Concepts of Chemistry

Important Formulae/Facts

- ***Mole concept :***
  - *Molar mass = mass of 1 mole*
  - *1 Mole = mass of* 6.023×10^{23} *particles (atom/molecule)*
  - *1 Mole atoms = gram atomic mass*
  - *1 g atom = N atoms =* 6.023×10^{23} *atoms*
  - *1 g molecule = N molecules =* 6.023×10^{23} *molecules = gram molecular mass = 22.4 L of any gas at STP*
  - *1 Mole ionic compound = Gram formula mass =* 6.023×10^{23} *formula units*
  - *Number of moles*
    $$= \frac{\text{Given number of particles } (N)}{\text{Avogadro's number of particles } (N_0)}$$
- *Expressing mass of a substance in terms of moles :*
  - *For elements :*
    $$\text{Number of moles} = \frac{\text{Given mass } (m)}{\text{Gram atomic mass } (M)}$$
  - *For compounds :*
    $$\text{Number of moles} = \frac{\text{Given mass } (m)}{\text{Gram molar mass } (M)}$$
- $\text{Number of molecules} = \dfrac{\text{Mass of the substance}}{\text{Molar mass}} \times N_0$
- $\text{Number of gram-atoms} = \dfrac{\text{Mass in grams}}{\text{Gram atomic mass}}$
- *Molecular mass = Mass of 22.4 L of vapour at STP*
- ***Conversion factors :***

1 m	= 39.37 inch
1 inch	= 2.54 cm
1 litre	= 1000 mL = 1000 cm^3
	= 10^{-3} m^3 = 1 dm^3
1 gallon	= 3.785412 L
1 lb	= 453.59237 g
1 N	= 1 kg m s^{-2}
1 J	= 1 N m = 1 kg m^2 s^{-2}
1 cal	= 4.184 J
	= 2.613×10^{19} eV
1 eV	= 1.602189×10^{-19} J
	= 3.827×10^{-20} cal
1 eV/atom	= 96.485 kJ mol^{-1}
1 amu	= $1.6605653 \times 10^{-27}$ kg
1 joule	= 10^7 erg
1 esu	= 3.3356×10^{-10} C
1 dyne	= 10^{-5} N
1 atm	= 101325 N m^{-2}
	= 101325 Pa
1 bar	= 1×10^5 N m^{-2}
1 litre atm	= 101.3 J = 24.21 cal
1 debye (D)	= 1×10^{-18} esu cm
1 mol of a gas	= 22.4 L *at STP*
1 mol of a substance	= N_0 *molecules*
1 g atom	= N_0 *atoms*
1 g cm^{-3}	= 1000 kg m^{-3}

- ***Law of Conservation of Mass***
  Matter can neither be created nor be destroyed.
- ***Law of Definite Proportions***
  A given compound always contains exactly the same proportion of elements by weight.
- ***Law of Multiple Proportions***
  If two elements can combine to form more than one compound, the masses of one element that combine with a fixed mass of the other element, are in the ratio of small whole numbers.
- ***Gay Lussac's Law of Gaseous Volumes***
  When gases combine or are produced in a chemical reaction they do so in a simple ratio by volume provided all gases are at same temperature and pressure.
- ***Avogadro's Law***
  Equal volumes of gases at the same temperature and pressure should contain equal number of molecules.
- ***Dalton's Atomic Theory***
  - *Matter consists of indivisible atoms.*
  - *All the atoms of a given element have identical properties including identical mass. Atoms of different elements differ in mass.*
  - *Compounds are formed when atoms of different elements combine in a fixed ratio.*
  - *Chemical reactions involve reorganisation of atoms. These can neither be created nor be destroyed in a chemical reaction.*

Important Notes

Introduction

Chemistry is that branch of science which deals primarily with the study of composition, properties and interaction of matter. These aspects can be described in and understood in terms of basic constituents of matter *i.e.*, atoms and molecules. That is why chemistry is also called the science of atoms and molecules.

Nature of Chemistry

- **Physical chemistry :** The branch of chemistry which deals with different laws and theories as well as the relations between physical properties of substances and their chemical composition and transformations.
- **Inorganic chemistry :** The branch of chemistry which deals with the synthesis, structures, composition and behaviour of inorganic compounds. This field covers all chemical compounds except the carbon based compounds usually containing C — H bond.
- **Organic chemistry :** The branch of chemistry which deals with the structures, properties and reactions of organic compounds which are composed of carbon and hydrogen and may contain some other elements.
- **Biochemistry :** The branch of chemistry which deals with the structure and behaviour of the components of cells and the chemical processes in living organisms.
- **Analytical chemistry :** The branch of chemistry which deals with the qualitative and quantitative analysis of different substances.

Matter

Anything that occupies spaces and possesses mass is called matter.

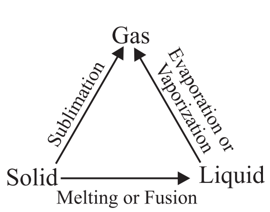
Physical Classification of Matter

Depending upon their physical characteristics, matter is generally classified as solids, liquids and gases, differentiating properties of which are tabulated below.

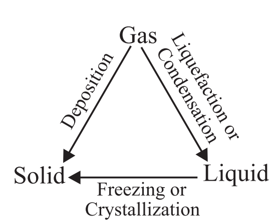
Differentiating properties of solids, liquids and gases

S. No.	Characteristic	Solids	Liquids	Gases
1.	Arrangement of particles	Particles are held very close to each other in an ordered manner.	Particles are close to each other but not so ordered as solids.	Particles are far apart as compared to that of solids and liquids.
2.	Movement of particles	Particles are not free to move.	Particles can move around.	The movement of particles is easy and fast.
3.	Compressibility and hardness	Hard and incompressible due to close packed structure.	More compressible due to more empty space in structure.	Most compressible due to large interparticle empty space.
4.	Volume	Definite	Definite	Not definite
5.	Shape	Definite	Not definite, take the shape of the container in which they are kept.	Not definite, completely occupy the container in which they are placed.
6.	Diffusion	No diffusion	Slow diffusion	Fast diffusion

- The three states of matter can be interconverted by changing the conditions of temperature and pressure as follows :

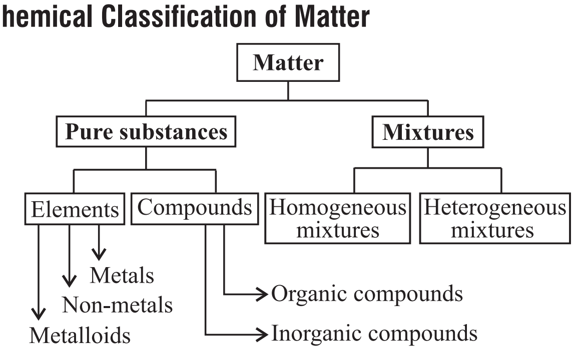


Endothermic state changes



Exothermic state changes

Chemical Classification of Matter



Pure Substances

A substance that consists of only a single type of constituent particles is called pure substance, *e.g.*, gold, water, etc. Based upon the nature of the constituent particles, a pure substance is of two types, *i.e.*, elements and compounds.

I. Elements

- An element is a basic form of matter that cannot be broken down into simpler substances by chemical reactions. An element is a pure substance. On the basis of variation in properties, elements can be broadly classified as metals, non-metals and metalloids.
- **Metals :** These are generally solids and have characteristics such as hardness, malleability, ductility, high tensile strength, lustre and ability to conduct heat and electricity. Some common examples are copper, iron, aluminium, silver, zinc, etc.
- **Non-metals :** These are generally, non-lustrous, brittle, poor conductors of heat and electricity. Some common examples are sulphur, phosphorus, nitrogen, hydrogen, oxygen, etc.
- **Metalloids :** These elements have characteristics common to metals as well as non-metals. Some common examples are arsenic, tin, bismuth, antimony, etc.

II. Compounds

- Compounds are defined as substances of definite compositions which can be decomposed into two or more substances by a simple chemical process. Water, sodium chloride, sugar, alcohol, carbon dioxide, acetic acid are some common compounds.
  - For example, decomposition of a compound water gives hydrogen and oxygen gases. Hydrogen gas is combustible. Oxygen helps combustion but water is used as fire extinguisher.

Mixtures

- Mixture always contains two or more substances which may be either homogeneous or heterogeneous.
  - Homogeneous mixture has uniform composition, appearance and properties throughout as the components completely mix with each other.
  - Heterogeneous mixture has two or more physically distinct phases and the composition is not uniform throughout.

Properties of Matter and Their Measurement

Every substance has unique or characteristic properties. These properties can be classified into two categories: physical properties and chemical properties.

- **Physical properties** are associated with physical existence of matter. These are inbuilt characteristics of a substance that can be determined without altering its composition *e.g.*, states of matter, colour, taste, odour, melting point, boiling point, density, etc.
- **Chemical property** is the ability of a substance to form new substances, either by decomposition or by reaction with other substances *e.g.*, acidity or basicity, combustibility, reactivity etc.

Measurement

In everyday life, we come across a number of measurements like kilograms (for mass), litres (for volume), metres (for length) etc. In addition to this common measurement we need to measure a number of other quantities as concentration, temperature, pressure, density, amount of electrical charge, etc.

All such quantities which we come across during the scientific studies are called physical quantities.

- **Units of measurement :** The value of physical quantity is always expressed in two parts : numerical value and unit.
- **Unit :** A unit is defined as the standard of reference chosen to measure any physical quantity.
  - **Fundamental units :** These are independent units and cannot be derived from any other units *e.g.*, units of mass (g or kg), length (cm or m) and time (s or min), etc.
  - **Derived units :** The units that can be derived from fundamental units are called derived units *e.g.*, units of density $\left(=\dfrac{\text{Mass (kg)}}{\text{Volume (m}^3)}\right)$, volume (= (length)^3 = m^3), etc.
- **SI units :** SI system of units or International System of Units was established by 11th General Conference on Weights and Measures and in French is called '*Le Systeme International d'Unites*.
  - The SI system has seven basic units :

Physical quantity	Symbol for quantity	Name of unit	Symbol for unit
Length	l	metre	m
Mass	m	kilogram	kg
Time	t	second	s
Temperature	T	kelvin	K
Electric current	I	ampere	A
Luminous intensity	I_v	candela	cd
Amount of substance	n	mole	mol

Some Derived SI Units

- Area = length × length = metre^2 = m^2
- Volume = length × breadth × height = m × m × m = m^3
- Speed = distance / time = m s^{-1}
- Acceleration $= \dfrac{\text{Change in velocity (m s}^{-1})}{\text{Time (s)}} = \text{m s}^{-2}$

- Force = mass (kg) × acceleration ($m\ s^{-2}$)
  $= kg\ m\ s^{-2}$ = Newton (N)
- Pressure = $\frac{\text{force (kg m s}^{-2})}{\text{area (m}^2)}$
  $= kg\ m^{-1}s^{-2}$ = Pascal (Pa)
- Energy = force ($kg\ m\ s^{-2}$) × distance (m)
  $= kg\ m^2s^{-2}$ = Joule (J)
- Density = mass/volume = $kg/m^3 = kg\ m^{-3}$
- Concentration $= \frac{\text{amount of solute}}{\text{volume of solution}} = \frac{\text{mol}}{\text{m}^3} = \text{mol m}^{-3}$

Some Physical Quantities

- **Mass (*m*)** is the amount of matter present in a substance. It remains constant for a substance at all the places. Its SI unit is kilogram (kg) but in laboratories usually gram (1 kg = 1000 g) is used.
- **Weight (*w*)** is the force exerted by gravity on an object. It varies from place to place due to change in gravity.
- **Volume (*v*) :** The space occupied by matter (usually by liquid or a gas) is called its volume.
  Volume = $(\text{length})^3 = m^3$
  Thus, SI unit of volume is m^3.
  1 L = 1000 mL = 1000 cm^3 = 1 dm^3
  1 $m^3 = (100\ cm)^3 = 10^6\ cm^3 = 10^3\ dm^3 = 10^3$ L
- **Density (*d*) :** It is defined as the amount or mass per unit volume and has units $kg\ m^{-3}$ or $g\ cm^{-3}$. (Here kg or g represents mass and m^3 or cm^3 represents volume).
  Density = Mass/Volume
- **Temperature (*T*) :** It is defined as the degree of hotness or coldness. There are three common scale to measure temperature: °C (degree Celsius), °F (degree Fahrenheit) and K (Kelvin).
  K is the SI unit. The temperature on two scales (°C and °F) are related to each other by the following relationship,
  $°F = \frac{9}{5}(°C) + 32$
  The kelvin scale is related to celsius scale as follows :
  K = °C + 273.15

Laws of Chemical Combination

Law of Conservation of Mass

- The law was first established by French chemist Lavoisier.
- This law states "in all chemical changes, the total mass of the system remains constant" or "in a chemical change, mass is neither created nor destroyed".

Law of Definite Proportions or Constant Composition

- This law was proposed by Proust.
- This law states "elements always combine in fixed ratio of their weights" or "a pure chemical compound contains the same elements in the fixed proportion of their weights regardless the mode of preparation".

Law of Multiple Proportions

- This law was established by Dalton.
- It states "if two elements form two or more than two compounds between them, then the masses of one of the elements which combine with a fixed mass of the other, bear a simple whole number ratio".

Gay Lussac Law of Gaseous Volumes

It states that when gases combine or are produced in a chemical reaction, they do so in a simple ratio by volume provided all gases are at same temperature and pressure.

Avogadro Law

Avogadro proposed that equal volumes of gases at the same temperature and pressure should contain equal number of molecules.

Dalton's Atomic Theory

Main postulates of Dalton's atomic theory are as follows :

- Matter consists of minute, indivisible, indestructible particles, called atoms.
- Atoms of an element are identical to each other. They have the same mass and size.
- Atoms of different elements differ in properties and have different masses and sizes.
- Compounds are formed when atoms of different elements combine with each other in simple numerical ratios and the relative numbers and kind of atoms are always the same in a given compound.
- Chemical reactions involve reorganisation of atoms. These are neither created nor destroyed in a chemical reaction.

The main drawbacks of Dalton's atomic theory are :

- It could explain the laws of chemical combination by mass but failed to explain the law of gaseous volumes.
- It could not explain why atoms of different elements have different masses, sizes, valencies, etc.
- It failed to explain the nature of forces that bind together atoms in a molecule.
- It could not explain how and why atoms of different elements combine with each other to form compound atoms or molecules.

Atomic and Molecular Masses

Atomic Mass

- Atomic mass of an element is a number which indicates as to how many times an atom of that element is heavier on an average, as compared with $1/12^{th}$ of the mass of an atom of carbon-12 ($_6C^{12}$).
  Since the atomic mass is a ratio, it has no unit. However, it is expressed in a.m.u. (atomic mass unit) or u (unified mass).
- **Atomic mass unit :** One atomic mass unit is defined as a mass exactly equal to one twelfth the mass of one carbon-12 atom.

$$1 \text{ a.m.u.} = \frac{1}{12} \times \text{ mass of C - 12 atom}$$

$$= \frac{1}{12} \times \frac{12 \text{ g}}{6.023 \times 10^{23}} = 1.66 \times 10^{-24} \text{ g}$$

Atomic mass of an element

$$= \frac{\text{Mass of one atom of the element}}{1 \text{ amu}}$$

- Gram atomic mass is the mass in gram of one mole of atoms in a monoatomic element. It is numerically equal to the atomic mass in amu.

  No. of gram atoms or mole atoms

$$= \frac{\text{Mass of an element in g}}{\text{Gram atomic mass}}$$

- **Average atomic mass :** Average atomic mass of an element can be calculated as

  Average atomic mass

$$= \frac{\text{R.A.}(1) \times \text{At. mass (1)} + \text{R.A. (2)} \times \text{At. mass (2)}}{\text{R.A. (1)} + \text{R.A. (2)}}$$

  Here R.A. = Relative abundance

Molecular Mass

- The average mass of a molecule of a substance in atomic mass unit is called its molecular mass (M).
- **Relative molecular mass :** The relative molecular mass of a substance is the average relative mass of its molecule as compared with the mass of a ^{12}C atom taken as 12 units.

  Relative molecular mass, M_r

$$= \frac{\text{Average mass of a molecule of the substance}}{\frac{1}{12} \times (\text{Mass of an atom of } ^{12}\text{C})}$$

  - The relative molecular mass (M_r) is a number and has no unit.

Formula Mass

- The formula mass is obtained by adding the atomic masses of all the atoms present in one formula unit of an ionic compound. For example,

  Formula mass of sodium chloride

  = Atomic mass of sodium + Atomic mass of chlorine

  = 23 u + 35.5 u = 58.5 u

Mole Concept and Molar Mass

- A mole (mol) is defined as the number of atoms in 12.00 g of carbon-12 isotope. The number of atoms in 12 g of carbon-12 has been found experimentally to be 6.023×10^{23}. This number is also known as Avogadro's number.
- The mass of one mole atoms of any element is exactly equal to the atomic mass in grams (gram-atomic mass or gram atom) of that element.
- One mole of any substance will have mass equal to formula mass of that substance expressed in grams.
- Number of moles of a substance

$$= \frac{\text{Mass of substance in gram}}{\text{Molecular mass of substance in gram}}$$

$$= \frac{\text{No. of particles}}{6.023 \times 10^{23}}$$

- Mass of one atom of an element

$$= \frac{\text{Gram atom of an element}}{6.023 \times 10^{23}}$$

- Mass of one molecule of a substance

$$= \frac{\text{Gram molecular mass of the substance}}{6.023 \times 10^{23}}$$

Moles and Gases

- One mole of any gas occupies 22.4 L of volume at STP.
- Number of moles of a gas (n)

$$\frac{\text{Volume of the gas at STP}}{\text{Molar volume of gas}} = \frac{\text{Volume of the gas at STP}}{22.4 \text{ dm}^3 \text{ mol}^{-1}}$$

- Number of molecules

$$= \frac{\text{Volume of gas in litres at NTP}}{22.4} \times 6.023 \times 10^{23}$$

Previous Years' Questions

1. What is the quantity of hydrogen gas liberated when 46 g sodium reacts with excess ethanol?
(Given At. mass of Na = 23)
(a) 2.4×10^{-3} kg (b) 2.0×10^{-3} kg
(c) 4.0×10^{-3} kg (d) 2.4×10^{-2} kg *(2017)*

2. What is the SI unit of density?
(a) g cm^{-3} (b) g m^{-3}
(c) kg m^{-3} (d) kg cm^{-3} *(2018)*

3. Which symbol replaces the unit of atomic mass, amu?
(a) u (b) A (c) M (d) n *(2018)*

4. The volume of 1 mole of any pure gas at standard temperature and pressure is always equal to
(a) 0.022414 m^3
(b) 22.414 m^3
(c) 2.2414 m^3
(d) 0.22414 m^3 *(2019)*

5. How many gram of sodium (atomic mass 23 u) is required to prepare one mole of ethane from methyl chloride by Wurtz reaction?
(a) 2 (b) 23
(c) 11.5 (d) 46 *(2019)*

6. What is the atomicity of aluminium phosphate?
(a) 8 (b) 6 (c) 5 (d) 13 *(2019)*

7. Which of the following sets of components form homogeneous mixture?
(a) Phenol + Water (b) Sugar + Benzene
(c) Silver chloride + Water (d) Ethyl alcohol + Water *(2019)*

8. What is the percentage of carbon in urea?
(At. mass : C = 12, H = 1, N = 14, O = 16)
(a) 20% (b) 26.6%
(c) 6.67% (d) 46.0% *(2019)*

9. Suppose the elements *X* and *Y* combine to form two compounds XY_2 and X_3Y_2 when 0.1 mole of XY_2 weighs 10 g and 0.05 mole of X_3Y_2 weighs 9 g, the atomic weights of *X* and *Y* are respectively
(a) 40 and 30 (b) 50 and 20
(c) 30 and 80 (d) 40 and 20 *(2020)*

10. Find the formula weight of reactants if the formula weight of product is 54 u and the percent atom economy is 75.
(a) 24 u (b) 30 u
(c) 72 u (d) 80 u *(2022)*

11. What is the amount of water formed by combustion of 1.6 g methane?
(a) 3.2 g (b) 16 g
(c) 6.2 g (d) 3.6 g *(2023)*

12. Calculate the percent atom economy when a product of formula weight 175 u is obtained in a chemical reaction using 225 u formula weight reactant.
(a) 70.1% (b) 77.7%
(c) 90.5% (d) 95.0% *(2023)*

13. What is number of atoms present in 2.24 dm^3 $NH_{3(g)}$ at STP?
(a) 6.022×10^{22} (b) 2.4088×10^{23}
(c) 1.8066×10^{22} (d) 6.022×10^{23} *(2023)*

14. Which of the following pair of compounds demonstrates the law of multiple proportions?
(a) CH_4, CCl_4 (b) BF_3, NH_3
(c) CO, CO_2 (d) NO_2, CO_2 *(2023)*

15. What is the number of moles of water molecules required to prepare *n* moles of methane from *n* moles of methyl magnesium iodide?
(a) *n* (b) 2*n* (c) 0.5*n* (d) 0.1*n* *(2023)*

16. What is the mass in gram of 1 atom of an element if its atomic mass is 10 u?
(a) 2.06056×10^{-22} g (b) 1.66056×10^{-23} g
(c) 1.06056×10^{-24} g (d) 3.66056×10^{-25} g *(2023)*

17. What volume of ammonia is formed when 10 dm^3 dinitrogen reacts with 30 dm^3 dihydrogen at same temperature and pressure?
(a) 30 dm^3 (b) 20 dm^3
(c) 15 dm^3 (d) 10 dm^3 *(2023)*

Hints & Explanations

1. (b) : $\underset{\text{Ethanol}}{2CH_3CH_2OH} + \underset{\text{Sodium}}{2Na} \longrightarrow \underset{\text{Sodium ethoxide}}{2CH_3CH_2ONa} + \underset{\text{Hydrogen}}{H_2}$

Molar mass of Na = 23 g mol^{-1}
46 g of Na or 2 moles of Na react with 2 moles of ethanol to produce one mole of hydrogen.
Mass of 1 mole of H_2 = 2 g = 2×10^{-3} kg

2. (c) **3. (a)** **4. (a)**

5. (d) : Wurtz reaction :

$$\underset{\text{Methyl chloride}}{2CH_3Cl} + 2Na \xrightarrow[-2NaCl]{\text{dry ether}} \underset{\text{Ethane}}{CH_3CH_3}$$

One mole of ethane requires 2 moles of sodium.
∴ 46 g of Na is required to prepare one mole of ethane.

6. (b) : The atomicity of aluminium phosphate ($AlPO_4$) is 6.

7. (d)

8. (a) : Urea (NH_2CONH_2),
Molar mass = 28 +12 +16 + 4 = 60
Mass of C in urea = 12
% of carbon = (12/60) × 100 = 20%

9. (a) : Let *x* and *y* be the atomic weights of *X* and *Y*.
Given, 0.1 mole of XY_2 weighs 10 g.

$$\Rightarrow x + 2y = \frac{10}{0.1} = 100 \quad \text{...(i)}$$

0.05 mole of X_3Y_2 weighs 9 g.

$$\Rightarrow 3x + 2y = \frac{9}{0.05} = 180 \quad \text{...(ii)}$$

From (i), $x = 100 - 2y$
Substituting the value of *x* in eq. (ii),
$3(100 - 2y) + 2y = 180$
$\Rightarrow 300 - 6y + 2y = 180 \Rightarrow y = 30,\ x = 100 - 2(30) = 40$
Hence, the atomic weights of *X* and *Y* are 40 g and 30 g respectively.

10. (c) : Given, percent atom economy is 75% and formula weight of product is 54 u.

Hence, formula weight of reactants $= \frac{100 \times 54}{75} = 72$ u

11. (d) : $\underset{\substack{\text{1 mole}\\ \text{or 16 g}}}{CH_{4(g)}} + 2O_{2(g)} \rightarrow CO_{2(g)} + \underset{\substack{\text{2 moles}\\ \text{or 36 g}}}{2H_2O_{(l)}}$

16 g methane on combustion produces 36 g water.
∴ 1.6 g methane on combustion produces 3.6 g water.

12. (b) : Percent atom economy

$$= \frac{\text{Formula weight of product}}{\text{Formula weight of reactant}} \times 100 = \frac{175 \times 100}{225} = 77.7\%$$

13. (b) : Number of atoms in 22.4 dm^3 = 4 × N_A

$$\text{Number of atoms in } 2.24\ dm^3 = \frac{4 \times N_A \times 2.24}{22.4}$$

$$= \frac{4 \times 6.022 \times 10^{23}}{10} = 2.4088 \times 10^{23}$$

14. (c) : Carbon and oxygen combine in a fixed ratio by mass to form CO (3 : 4) and CO_2 (3 : 8).

15. (a) : Methyl magnesium iodide reacts with water to produce methane as per the given reaction,

$$\underset{1\text{ mol}}{CH_3MgI} + \underset{1\text{ mol}}{H_2O} \rightarrow \underset{1\text{ mol}}{CH_4}$$

Thus, *n* mole of methyl magnesium iodide will react with *n* mole of water to produce *n* mole of methane.

16. (b) : 1 mole of an element contains 6.022 × 10^{23} atoms. 6.022 × 10^{23} atoms weigh 10 u or 10 g.

$$\text{Mass of 1 atom} = \frac{10}{6.022 \times 10^{23}} = 1.66056 \times 10^{-23}\ \text{g}$$

17. (b) : $N_2 + 3H_2 \rightleftharpoons 2NH_3$

From the balanced chemical equation, it can be concluded that 1 volume of nitrogen produces 2 volume of ammonia *i.e.*, 10 dm^3 of nitrogen produces 20 dm^3 of ammonia. Or, 3 volume of hydrogen produces 2 volume of ammonia *i.e.*, 30 dm^3 of hydrogen produces 20 dm^3 of ammonia.

Solved Examples

Example 1 (a) Two substances *X* and *Y* combine to give a substance *Z*. The process is exothermic and *Z* has properties different from those of *X* and *Y*. Is the substance *Z* an element, a mixture or a compound?

Give explanation to support your answer.

(b) List any three differences between a mixture and a compound.

Soln.: (a) The substance *Z* is a compound. This is because

(i) heat is evolved during the formation of *Z*.

(ii) the properties of *Z* are different from those of *X* and *Y*.

(b)

Mixture		Compound	
1.	The constituents of a mixture may be present in any ratio.	1.	The constituents of a compound are always present in a fixed ratio by mass.
2.	Mixture may or may not be homogeneous in nature.	2.	Compounds are always homogeneous in nature.
3.	The properties of a mixture are midway between those of its constituents.	3.	The properties of a compound are entirely different from those of its constituents.

Example 2 What do you mean by fundamental units and derived units and give two examples of each?

Soln.: Fundamental units : These are independent units and cannot be derived from any other units, *e.g.*, units of mass (g or kg) and length (cm or m).

Derived units : The units that can be derived from fundamental units are called derived units *e.g.*, unit of density $\left(=\frac{\text{Mass (kg)}}{\text{Volume }(m^3)}\right)$ is kg m^{-3} ;

unit of volume = $(\text{length})^3 = m^3$

Example 3 Carbon and oxygen are known to form two compounds. The carbon content in one of these compounds is 42.9% while in the other, it is 27.3%. Show that this data is in the agreement with the law of multiple proportions.

Soln.:

Compound I	**Compound II**
Mass of carbon = 42.9 g	Mass of carbon = 27.3 g
Mass of oxygen = 100 – 42.9 = 57.1 g	Mass of oxygen = 100 – 27.3 = 72.7 g
∵ Mass of carbon which combines with 57.1 g of oxygen = 42.9 g	∵ Mass of carbon which combines with 72.7 g of oxygen = 27.3 g

∵ Mass of carbon which combines with 1.0 g of oxygen in compound I $= \frac{42.9}{57.1} = 0.75$ g

∴ Mass of carbon which combines with 1.0 g of oxygen in compound II $= \frac{27.3}{72.7} = 0.375$ g

Thus, The ratio of mass of carbon combining with fixed mass of oxygen is 0.75 : 0.375 or 2 : 1.

This is a simple ratio and therefore explains the law of multiple proportions.

Example 4 (i) Copper sulphate crystals contain 25.45% Cu and 36.07% H_2O. If the law of constant proportions is true, then calculate the weight of Cu required to obtain 40 g of crystalline copper sulphate.

(ii) 1.375 g of cupric oxide on reduction in hydrogen gas gives 1.098 g of copper. In another experiment, 1.179 g of metallic copper produced 1.476 g of copper oxide. Show that these results illustrate the law of constant (or definite) proportions.

Soln.: (i) For 100 g of $CuSO_4.5H_2O$, required amount of Cu is 25.45 g.

For 40 g of $CuSO_4.5H_2O$, required amount of Cu is $\frac{25.45}{100} \times 40 = 10.18$ g

(ii) Experiment 1 :

Mass of copper oxide taken = 1.375 g

Mass of copper obtained = 1.098 g

Therefore, mass % of copper present in copper oxide

$$= \frac{1.098 \times 100}{1.375} = 79.86\%$$

Experiment 2 :
Mass of copper oxide produced = 1.476 g
Mass of copper used = 1.179 g
Therefore, mass % of copper present in copper oxide

$$= \frac{1.179 \times 100}{1.476} = 79.89\%$$

Since, the percentage of copper in the two samples of copper oxide is the same (within ± 0.5%), hence the law of definite proportions is verified.

Example 5 (a) What is an atom according to Dalton's atomic theory?
(b) One of the statements of Dalton's atomic theory is given below :
"Compounds are formed when atoms of different elements combine in a fixed ratio."
Name the laws which support the above statement and which do not.

Soln.: (a) According to Dalton's atomic theory, an atom is the ultimate particle of matter which cannot be further divided.
(b) Law of definite proportions and law of multiple proportions support the given statement while law of conservation of mass and Avogadro's law do not support.

Example 6 Calculate the mass of a single atom of sulphur and a single molecule of carbon dioxide.

Soln.: Gram atomic mass of sulphur = 32 g
Mass of one sulphur atom

$$= \frac{\text{Gram atomic mass}}{6.023 \times 10^{23}} = \frac{32}{6.023 \times 10^{23}} = 5.31 \times 10^{-23}\ \text{g}$$

Formula of carbon dioxide = CO_2
Molecular mass of CO_2 = 12 + 2 × 16 = 44
Gram-molecular mass of CO_2 = 44 g
Mass of one molecule of CO_2

$$= \frac{\text{Gram molecular mass}}{6.023 \times 10^{23}} = \frac{44}{6.023 \times 10^{23}} = 7.308 \times 10^{-23}\ \text{g}$$

Example 7 (i) Calculate :
(a) mass of 2.5 gram atoms of magnesium.
(b) gram atoms in 1.4 grams of nitrogen (atomic mass of Mg = 24, N = 14).
(ii) What is the molecular mass of a substance, each molecule of which contains 9 atoms of carbon, 13 atoms of hydrogen and 2.33×10^{-23} g of other component?

Soln.: (i) (a) 1 gram atom of Mg = 24 g
2.5 gram atoms of Mg = 24 × 2.5 = 60 g
(b) 1 gram atom of N = 14 g

$$1.4 \text{ g of N} = \frac{1}{14} \times 1.4 = 0.1 \text{ gram atom.}$$

(ii) Mass of 9 atoms of carbon = 9 × 12 amu = 108 u
Mass of 13 atoms of hydrogen = 13 × 1 amu = 13 u
Mass of 2.33×10^{-23} g of other component

$$= (1\ \text{u}) \times \frac{(2.33 \times 10^{-23}\ \text{g})}{(1.66 \times 10^{-24}\ \text{g})} = 14.04\ \text{u}$$

Molecular mass of the substance = (108 + 13 + 14.04) u
= 135.04 u

Example 8 (i) How many electrons are there in 0.3 g water?
(ii) Calculate the number of moles in the following :
(a) 7.85 g iron (b) 4.68 mg of silicon (c) 65.6 μg of carbon.

Soln.: (i) Number of molecules in 0.3 g water

$$= \frac{0.3}{18} \times 6.023 \times 10^{23} = 10^{22}$$

One molecule of water contains 10 electrons.
⇒ Number of electrons in 0.3 g water = $10 \times 10^{22} = 10^{23}$
(ii) (a) In 7.85 g of iron

$$\text{Moles of iron} = \frac{\text{Mass of iron}}{\text{Atomic mass}} = \frac{7.85}{55.8} = 0.141 \text{ mol}$$

(b) 4.68 mg of silicon = 4.68×10^{-3} g of silicon

$$\text{Moles of silicon} = \frac{\text{Mass of silicon}}{\text{Atomic mass}} = \frac{4.68 \times 10^{-3}}{28.1}$$

$= 1.66 \times 10^{-4}$ mol
(c) 65.6 μg of carbon = 65.6×10^{-6} g of carbon

$$\text{Moles of carbon} = \frac{\text{Mass of carbon}}{\text{Atomic mass}} = \frac{65.6 \times 10^{-6}}{12}$$

$= 5.47 \times 10^{-6}$ mol

Example 9 Calculate the number of atoms present in 11.5 litres of H_2 at N.T.P.

Soln.: Number of atoms present in 22.4 litres of H_2 at N.T.P. = $2 \times 6.023 \times 10^{23}$
∴ Number of atoms present in 11.5 litres of H_2 at N.T.P.

$$= \frac{2 \times 6.023 \times 10^{23}}{22.4} \times 11.5 = 6.18 \times 10^{23}$$

Example 10 Calculate the number of Cl^- and Ca^{2+} ions in 222 g of anhydrous $CaCl_2$.
Soln.: 1 mol of $CaCl_2$ = 40 + (35.5 × 2) = 111 g

$$\underset{1\text{ mol}}{CaCl_2} \longrightarrow \underset{1\text{ mol}}{Ca^{2+}} + \underset{2\text{ mol}}{2Cl^-}$$

111 g of $CaCl_2$ = 1 mol of Ca^{2+}
222 g of $CaCl_2$ = 2 mol of Ca^{2+}
111 g of $CaCl_2$ = 2 mol of Cl^-

$$222 \text{ g of } CaCl_2 = \frac{2}{111} \times 222 = 4 \text{ mol of } Cl^-$$

Therefore, number of Ca^{2+} ions
$= 2 \times N_A = 2 \times 6.023 \times 10^{23}$
$= 12.046 \times 10^{23}$
Number of Cl^- ions = $4 \times N_A = 4 \times 6.023 \times 10^{23}$
$= 24.092 \times 10^{23}$

Daily Practice Problems

Introduction

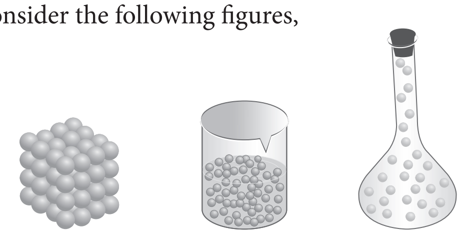
1. What are the basic constituents of matter?
(a) Atoms and molecules (b) Atoms and moles
(c) Molecules and ions (d) Nuclei and ions

Nature of Chemistry

2. Which one of the following is not a mixture?
(a) Air (b) Distilled water
(c) Gasoline
(d) Liquefied petroleum gas (LPG)

3. Iodized common salt is
(a) homogeneous mixture (b) heterogeneous mixture
(c) pure substance (d) oxidized substance.

4. Consider the following figures,



Solid I Liquid II Gas III

Choose the correct statement about I, II and III.
(a) I and II have definite volume but III does not have this property.
(b) I, II and III are interconvertible by changing the conditions of temperature and pressure.
(c) In the particles of I, freedom of movement is large.
(d) Both (a) and (b).

Properties of Matter and Their Measurement

5. SI unit of temperature is
(a) Kelvin (b) °C (c) °F (d) Dalton

6. 1 L = ________ m^3
(a) 1000 (b) 1 (c) 0.001 (d) 10

7. What is the human body temperature in Fahrenheit?
(a) 212 (b) 98.6 (c) 273.15 (d) 32

Laws of Chemical Combination

8. Law of constant composition does not hold good for
(a) exothermic compounds
(b) stoichiometric compounds
(c) endothermic compounds
(d) non-stoichiometric compounds.

9. 45.4 L of dinitrogen reacted with 22.7 L of dioxygen and 45.4 L of nitrous oxide was formed. The reaction is given below :
$2N_{2(g)} + O_{2(g)} \rightarrow 2N_2O_{(g)}$
Which law is being obeyed in this experiment?
(a) Gay Lussac's law
(b) Law of definite proportion
(c) Law of multiple proportion
(d) Avogadro's law

10. The following data are obtained when dinitrogen and dioxygen react together to form different compounds:

Mass of dinitrogen	Mass of dioxygen
28 g	16 g
14 g	16 g
28 g	48 g
28 g	80 g

Which law of chemical combination is obeyed by the above experimental data?
(a) Law of conservation of mass
(b) Law of definite proportions
(c) Law of multiple proportions
(d) Avogadro's Law

11. Which of the following statements best explains the law of conservation of mass?
(a) 100 g of water is heated to give steam.
(b) A sample of N_2 gas is heated at constant pressure without any change in mass.
(c) 36 g of carbon combines with 32 g of oxygen to form 68 g of CO_2.
(d) 10 g of carbon is heated in vacuum without any change in mass.

12. In an experiment, 2.4 g of iron oxide on reduction with hydrogen gave 1.68 g of iron. In another experiment, 2.7 g of iron oxide gave 1.89 g of iron on reduction. Which law is illustrated from the above data?
(a) Law of constant proportions
(b) Law of multiple proportions
(c) Gay Lussac's law of gaseous volume
(d) Law of conservation of mass

Dalton's Atomic Theory

13. Which of the following postulates of Dalton's atomic theory explains the law of constant proportion?
(a) Atoms of given element are identical in mass and chemical properties.
(b) Atoms combine in the ratio of small whole numbers to form compounds.
(c) The relative number and kind of atoms are constant in a given compound.
(d) All of these.

14. According to Dalton's atomic theory chemical reaction involve
(a) destruction of atoms (b) reorganisation of atoms
(c) reorganisation of nuclei (d) construction of atoms.

Atomic and Molecular Masses

15. Atomic mass unit = ________ kg.
(a) 1.66×10^{-27} kg (b) 1.66×10^{-28} kg
(c) 1.66×10^{-29} kg (d) 1.66×10^{-30} kg

16. The reference standard used for defining atomic mass is
(a) H-1 (b) C-12 (c) C-13 (d) C-14

17. For every one ^{37}Cl isotope, there are three ^{35}Cl isotopes in a sample of chlorine. What will be the average atomic mass of chlorine?
(a) 35 (b) 37 (c) 35.5 (d) 35.6

18. Atomic masses of elements are usually fractional because
(a) they exist as a mixture of isotopes of different masses
(b) they contain impurities of other atoms
(c) they are mixtures of isobars
(d) they cannot be weighed accurately.

19. The mass of 4 g-atom of S is
(a) 32 g (b) 128 g (c) 64 g (d) 256 g

20. A compound possesses 8% sulphur by mass. The least molecular mass is
(a) 200 (b) 400 (c) 155 (d) 355

21. 52 u of He contains
(a) 6 atoms (b) 13 atoms
(c) 13 atoms (d) 4 atoms.

22. The formula mass of MgO is
(a) 20 (b) 16 (c) 43 (d) 40

23. The molecular mass of glucose ($C_6H_{12}O_6$) is
(a) 160.8 u (b) 180.162 u
(c) 216.4 u (b) 134.821 u

24. 3 atoms of metal *M* combine with 2 atoms of nitrogen to give a compound containing 28% nitrogen and 72% of metal. What is the atomic weight of metal?
(a) 36 (b) 24 (c) 12 (d) 48

Mole Concept and Molar Mass

25. How many oxygen atoms will be present in 88 g of CO_2?
(a) 24.08×10^{23} (b) 6.023×10^{23}
(c) 44×10^{23} (d) 22×10^{24}

26. What is the total number of electrons present in 1.6 g of methane?
(a) 6.023×10^{23} (b) 16
(c) 12.04×10^{23} (d) 6.023×10^{24}

27. Which is heaviest?
(a) 25 g mercury (b) 2 mol carbon dioxide
(c) 2 mol water (d) 4 g atom of oxygen

28. Which of the following weighs the least?
(a) 2 g atom of N (b) 3×10^{23} atoms of C
(c) 1 mole of S (d) 7 g silver

29. How many atoms are contained in one mole of sucrose ($C_{12}H_{22}O_{11}$)?
(a) $45 \times 6.02 \times 10^{23}$ atoms/mol
(b) $20 \times 6.02 \times 10^{23}$ atoms/mol
(c) $5 \times 6.02 \times 10^{23}$ atoms/mol
(d) None of these

30. Rearrange the following (I to IV) in the order of increasing masses.
I. 1 molecule of oxygen
II. 1 atom of nitrogen
III. 1×10^{-10} g molecular weight of oxygen
IV. 1×10^{-8} g atomic weight of copper
(a) II < I < III < IV (b) IV < III < II < I
(c) II < III < I < IV (d) III < IV < I < II

31. The number of oxygen atoms in 4.4 g of CO_2 is
(a) 1.2×10^{23} (b) 6×10^{22}
(c) 6×10^{23} (d) 12×10^{23}

32. Number of moles of 1 m^3 gas at NTP is
(a) 44.6 (b) 40.6
(c) 42.6 (d) 48.6

33. How many gram of H_2SO_4 are present in 0.25 mole of H_2SO_4?
(a) 2.45 (b) 24.5
(c) 0.245 (d) 0.25

34. The number of moles of $BaCO_3$ which contains 1.5 moles of oxygen atoms is
(a) 0.5 (b) 1
(c) 3 (d) 6.023×10^{23}

35. If N_A is Avogadro's number then number of valence electrons in 4.2 g of nitride ions, N^{3-} is
(a) 1.6 N_A (b) 2.4 N_A
(c) 4.2 N_A (d) 3.2 N_A

36. The number of electrons in a mole of hydrogen molecule is
(a) 6.02×10^{23} (b) 12.046×10^{23}
(c) 3.0115×10^{23} (d) indefinite.

37. No. of charge on 1 mole of N^{3-} ion is
(a) 96500 C (b) 1.6×10^{19} C
(c) 3×96500 C (d) $3 \times 10 \times 10^{-19}$ C

38. The number of molecules of water in 333 g of $Al_2(SO_4)_3 \cdot 18H_2O$ is
(a) $18.0 \times 6.023 \times 10^{23}$ (b) $9.0 \times 6.023 \times 10^{23}$
(c) 18.0 (d) 36.0

39. How many atoms in total are present in 1 kg of sugar?
(a) 7.92×10^{25} atoms (b) 6×10^{23} atoms
(c) 6.022×10^{25} atoms (d) 1000 atoms

40. How many moles of electrons weigh one kilogram?
(a) 6.023×10^{23} (b) $\frac{1}{9.108} \times 10^{31}$
(c) $\frac{6.023}{9.108} \times 10^{54}$ (d) $\frac{1}{9.108 \times 6.023} \times 10^{8}$

41. The mass of one molecule of water is approximately
(a) 3×10^{-23} g (b) 18 g
(c) 1.5×10^{-23} g (d) 4.5×10^{-23} g

42. Which of the following has maximum number of molecules?
(a) 7 g N_2 (b) 2 g H_2
(c) 16 g NO_2 (d) 16 g O_2

43. Number of atoms in 4.25 g of NH_3 is nearly
(a) 1×10^{23} (b) 1.5×10^{23}
(c) 2×10^{23} (d) 6×10^{23}

44. Number of atoms in 558.5 gram Fe (at. wt. of Fe = 55.85 g mol^{-1}) is
(a) twice that in 60 g carbon
(b) 6.023×10^{22}
(c) half that in 8 g He
(d) $558.5 \times 6.023 \times 10^{23}$

45. Match the mass of elements given in column I with the no. of moles given in column II and mark the appropriate choice.

Column I		Column II	
(A)	28 g of He	(i)	2 moles
(B)	46 g of Na	(ii)	7 moles
(C)	60 g of Ca	(iii)	1 mole
(D)	27 g of Al	(iv)	1.5 moles

(a) (A) → (iv), (B) → (iii), (C) → (ii), (D) → (i)
(b) (A) → (i), (B) → (iii), (C) → (ii), (D) → (iv)
(c) (A) → (iii), (B) → (ii), (C) → (i), (D) → (iv)
(d) (A) → (ii), (B) → (i), (C) → (iv), (D) → (iii)

Moles and Gases

46. If 300 mL of a gas weighs 0.368 g at STP, what is its molecular weight?
(a) 30.16 g (b) 2.55 g (c) 27.5 g (d) 37.5 g

47. Number of moles of 1 m^3 gas at NTP is
(a) 44.6 (b) 40.6 (c) 42.6 (d) 48.6

48. The maximum number of molecules is present in
(a) 15 L of H_2 gas at STP (b) 5 L of N_2 gas at STP
(c) 0.5 g of H_2 gas (d) 10 g of O_2 gas.

49. What will be the standard molar volume of He, if its density is 0.1784 g/L at STP?
(a) 11.2 L (b) 22.4 L (c) 5.6 L (d) 2.8 L

50. How many number of molecules and atoms respectively are present in 2.8 litres of a diatomic gas at STP?
(a) 6.023×10^{23}, 7.5×10^{23}
(b) 6.023×10^{23}, 15×10^{22}
(c) 7.5×10^{22}, 15×10^{22}
(d) 15×10^{22}, 7.5×10^{23}

ANSWER KEYS

1. (a)	**2.** (b)	**3.** (a)	**4.** (d)	**5.** (a)	**6.** (c)	**7.** (b)	**8.** (d)	**9.** (a)	**10.** (c)
11. (c)	**12.** (a)	**13.** (b)	**14.** (b)	**15.** (a)	**16.** (b)	**17.** (c)	**18.** (a)	**19.** (b)	**20.** (b)
21. (b)	**22.** (d)	**23.** (b)	**24.** (b)	**25.** (a)	**26.** (a)	**27.** (d)	**28.** (b)	**29.** (a)	**30.** (a)
31. (a)	**32.** (a)	**33.** (b)	**34.** (a)	**35.** (b)	**36.** (b)	**37.** (c)	**38.** (b)	**39.** (a)	**40.** (d)
41. (a)	**42.** (b)	**43.** (d)	**44.** (a)	**45.** (d)	**46.** (c)	**47.** (a)	**48.** (a)	**49.** (b)	**50.** (c)

MATHEMATICS

CHAPTER 1

Trigonometry-II

Important Formulae/Facts

- ***Trigonometric Functions of Allied Angles***

Allied Angles \ Trigonometric Functions	sin	cos	tan	cot	sec	cosec
$-\theta$	$-\sin\theta$	$\cos\theta$	$-\tan\theta$	$-\cot\theta$	$\sec\theta$	$-\operatorname{cosec}\theta$
$\left(\frac{\pi}{2}\pm\theta\right)$	$\cos\theta$	$\mp\sin\theta$	$\mp\cot\theta$	$\mp\tan\theta$	$\mp\operatorname{cosec}\theta$	$\sec\theta$
$(\pi\pm\theta)$	$\mp\sin\theta$	$-\cos\theta$	$\pm\tan\theta$	$\pm\cot\theta$	$-\sec\theta$	$\mp\operatorname{cosec}\theta$
$\left(\frac{3\pi}{2}\pm\theta\right)$	$-\cos\theta$	$\pm\sin\theta$	$\mp\cot\theta$	$\mp\tan\theta$	$\pm\operatorname{cosec}\theta$	$-\sec\theta$
$(2\pi\pm\theta)$	$\pm\sin\theta$	$\cos\theta$	$\pm\tan\theta$	$\pm\cot\theta$	$\sec\theta$	$\pm\operatorname{cosec}\theta$

- $\sin(A+B)=\sin A\cos B+\cos A\sin B$
- $\cos(A+B)=\cos A\cos B-\sin A\sin B$
- $\sin(A-B)=\sin A\cos B-\cos A\sin B$
- $\cos(A-B)=\cos A\cos B+\sin A\sin B$
- $\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$
- $\tan(A-B)=\frac{\tan A-\tan B}{1+\tan A\tan B}$
- $\cot(A+B)=\frac{\cot A\cot B-1}{\cot A+\cot B}$
- $\cot(A-B)=\frac{\cot A\cot B+1}{\cot B-\cot A}$
- $\sin 2A=2\sin A\cos A=\frac{2\tan A}{1+\tan^2 A}$
- $\cos 2A=\cos^2A-\sin^2A=1-2\sin^2A=2\cos^2A-1$
  $=\frac{1-\tan^2 A}{1+\tan^2 A}$
- $\tan 2A=\frac{2\tan A}{1-\tan^2 A}$
- $\sin 3A=3\sin A-4\sin^3 A$
- $\cos 3A=4\cos^3 A-3\cos A$
- $\tan 3A=\frac{3\tan A-\tan^3 A}{1-3\tan^2 A}$
- $\sin A=2\sin\left(\frac{A}{2}\right)\cos\left(\frac{A}{2}\right)=\frac{2\tan(A/2)}{1+\tan^2(A/2)}$
- $\cos A=\cos^2\left(\frac{A}{2}\right)-\sin^2\left(\frac{A}{2}\right)=1-2\sin^2\left(\frac{A}{2}\right)$
  $=2\cos^2\left(\frac{A}{2}\right)-1=\frac{1-\tan^2 A/2}{1+\tan^2 A/2}$
- $\tan A=\frac{2\tan(A/2)}{1-\tan^2(A/2)}$
- $\cos A+\cos B=2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$
- $\cos A-\cos B=2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{B-A}{2}\right)$
- $\sin A+\sin B=2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$
- $\sin A-\sin B=2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$
- $2\sin A\cos B=\sin(A+B)+\sin(A-B)$
- $2\cos A\sin B=\sin(A+B)-\sin(A-B)$
- $2\cos A\cos B=\cos(A+B)+\cos(A-B)$
- $2\sin A\sin B=\cos(A-B)-\cos(A+B)$
- *If A, B, C are angles of a triangle ABC, then*
  - $\sin(A+B)=\sin(\pi-C)=\sin C$
  - $\sin(B+C)=\sin(\pi-A)=\sin A$
  - $\sin(C+A)=\sin(\pi-B)=\sin B$
  - $\sin\left(\frac{A+B}{2}\right)=\sin\left(\frac{\pi}{2}-\frac{C}{2}\right)=\cos\left(\frac{C}{2}\right)$

- $\sin\left(\frac{A+C}{2}\right)=\sin\left(\frac{\pi}{2}-\frac{B}{2}\right)=\cos\left(\frac{B}{2}\right)$
- $\sin\left(\frac{B+C}{2}\right)=\sin\left(\frac{\pi}{2}-\frac{A}{2}\right)=\cos\left(\frac{A}{2}\right)$
- $\cos(A+B)=\cos(\pi-C)=-\cos C$
- $\cos(B+C)=\cos(\pi-A)=-\cos A$
- $\cos(C+A)=\cos(\pi-B)=-\cos A$
- $\cos\left(\frac{A+B}{2}\right)=\cos\left(\frac{\pi}{2}-\frac{C}{2}\right)=\sin\frac{C}{2}$
- $\cos\left(\frac{B+C}{2}\right)=\cos\left(\frac{\pi}{2}-\frac{A}{2}\right)=\sin\frac{A}{2}$
- $\cos\left(\frac{C+A}{2}\right)=\cos\left(\frac{\pi}{2}-\frac{B}{2}\right)=\sin\frac{B}{2}$

Important Notes

Trigonometric Functions of sum and Difference of Angles

- $\sin(A+B)=\sin A\cos B+\cos A\sin B$
- $\sin(A-B)=\sin A\cos B-\cos A\sin B$
- $\cos(A+B)=\cos A\cos B-\sin A\sin B$
- $\cos(A-B)=\cos A\cos B+\sin A\sin B$
- $\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$
- $\tan(A-B)=\frac{\tan A-\tan B}{1+\tan A\tan B}$
- $\cot(A+B)=\frac{\cot A\cot B-1}{\cot A+\cot B}$
- $\cot(A-B)=\frac{\cot A\cot B+1}{\cot B-\cot A}$

Trigonometric Functions of Allied Angles

- Two angles are said to be allied if their sum or their difference is a multiple of $\pi/2$.
- The trigonometric function changes at allied angles which are given in the following table :

Allied Angles \ Trigonometric Functions	sin	cos	tan	cot	sec	cosec
$-\theta$	$-\sin\theta$	$\cos\theta$	$-\tan\theta$	$-\cot\theta$	$\sec\theta$	$-\operatorname{cosec}\theta$
$\left(\frac{\pi}{2}\pm\theta\right)$	$\cos\theta$	$\mp\sin\theta$	$\mp\cot\theta$	$\mp\tan\theta$	$\mp\operatorname{cosec}\theta$	$\sec\theta$
$(\pi\pm\theta)$	$\mp\sin\theta$	$-\cos\theta$	$\pm\tan\theta$	$\pm\cot\theta$	$-\sec\theta$	$\mp\operatorname{cosec}\theta$
$\left(\frac{3\pi}{2}\pm\theta\right)$	$-\cos\theta$	$\pm\sin\theta$	$\mp\cot\theta$	$\mp\tan\theta$	$\pm\operatorname{cosec}\theta$	$-\sec\theta$
$(2\pi\pm\theta)$	$\pm\sin\theta$	$\cos\theta$	$\pm\tan\theta$	$\pm\cot\theta$	$\sec\theta$	$\pm\operatorname{cosec}\theta$

Trigonometric Functions of Multiples Angles

- Number like $2A$, $3A$ and $A/2$ etc. are called multiples of A.
- For any angle A:
  - $\sin 2A=2\sin A\cos A=\frac{2\tan A}{1+\tan^2 A}$
  - $\cos 2A=\cos^2 A-\sin^2 A=1-2\sin^2 A=2\cos^2 A-1$ $=\frac{1-\tan^2 A}{1+\tan^2 A}$
  - $\tan 2A=\frac{2\tan A}{1-\tan^2 A}$
  - $\sin 3A=3\sin A-4\sin^3 A$
  - $\cos 3A=4\cos^3 A-3\cos A$
  - $\tan 3A=\frac{3\tan A-\tan^3 A}{1-3\tan^2 A}$
  - $\sin A=2\sin\frac{A}{2}\cos\frac{A}{2}$
  - $\cos A=\cos^2\frac{A}{2}-\sin^2\frac{A}{2}=2\cos^2\frac{A}{2}-1=1-2\sin^2\frac{A}{2}$
  - $\sin A=\frac{2\tan\frac{A}{2}}{1+\tan^2\frac{A}{2}}$
  - $\cos A=\frac{1-\tan^2\frac{A}{2}}{1+\tan^2\frac{A}{2}}$

- $\tan A = \dfrac{2\tan\frac{A}{2}}{1-\tan^2\frac{A}{2}}$

Factorization Formulae

It is conversion of the product of two trigonometric functions into sum or difference of two trigonometric functions of vise-versa.

- **Formulae for Conversion of Sum or Difference into Product**
- For any angles A and B :
  - $\cos A + \cos B = 2\cos\left(\dfrac{A+B}{2}\right)\cos\left(\dfrac{A-B}{2}\right)$
  - $\cos A - \cos B = 2\sin\left(\dfrac{A+B}{2}\right)\sin\left(\dfrac{B-A}{2}\right)$
  - $\sin A + \sin B = 2\sin\left(\dfrac{A+B}{2}\right)\cos\left(\dfrac{A-B}{2}\right)$
  - $\sin A - \sin B = 2\cos\left(\dfrac{A+B}{2}\right)\sin\left(\dfrac{A-B}{2}\right)$
- **Formulae for Conversion of Product into Sum or Difference**
  - $2\sin A\cos B = \sin(A+B) + \sin(A-B)$
  - $2\cos A\sin B = \sin(A+B) - \sin(A-B)$
  - $2\cos A\cos B = \cos(A+B) + \cos(A-B)$
  - $2\sin A\sin B = \cos(A-B) - \cos(A+B)$

Trigonometric Functions of Angles of a Triangle

- If A, B, C are angles of a triangle ABC, then
  - $\sin(A+B) = \sin(\pi - C) = \sin C$
  - $\sin(B+C) = \sin(\pi - A) = \sin A$
  - $\sin(C+A) = \sin(\pi - B) = \sin B$
  - $\sin\left(\dfrac{A+B}{2}\right) = \sin\left(\dfrac{\pi}{2}-\dfrac{C}{2}\right) = \cos\left(\dfrac{C}{2}\right)$
  - $\sin\left(\dfrac{A+C}{2}\right) = \sin\left(\dfrac{\pi}{2}-\dfrac{B}{2}\right) = \cos\left(\dfrac{B}{2}\right)$
  - $\sin\left(\dfrac{B+C}{2}\right) = \sin\left(\dfrac{\pi}{2}-\dfrac{A}{2}\right) = \cos\left(\dfrac{A}{2}\right)$
  - $\cos(A+B) = \cos(\pi - C) = -\cos C$
  - $\cos(B+C) = \cos(\pi - A) = -\cos A$
  - $\cos(C+A) = \cos(\pi - B) = -\cos A$
  - $\cos\left(\dfrac{A+B}{2}\right) = \cos\left(\dfrac{\pi}{2}-\dfrac{C}{2}\right) = \sin\dfrac{C}{2}$
  - $\cos\left(\dfrac{B+C}{2}\right) = \cos\left(\dfrac{\pi}{2}-\dfrac{A}{2}\right) = \sin\dfrac{A}{2}$
  - $\cos\left(\dfrac{C+A}{2}\right) = \cos\left(\dfrac{\pi}{2}-\dfrac{B}{2}\right) = \sin\dfrac{B}{2}$

Note :

- $\sin n\pi = 0$, $\cos n\pi = (-1)^n$
- $\sin(n\pi + \theta) = (-1)^n \sin\theta$
- $\cos(n\pi + \theta) = (-1)^n \cos\theta$
- $\sin(n\pi - \theta) = (-1)^{n-1}\sin\theta$
- $\cos(n\pi - \theta) = (-1)^n \cos\theta$
- $\sin\left(\dfrac{n\pi}{2}+\theta\right) = (-1)^{\frac{n-1}{2}}\cos\theta$, if n is odd

  $= (-1)^{\frac{n}{2}}\cos\theta$, if n is even
- $\left|\sin\dfrac{A}{2}+\cos\dfrac{A}{2}\right| = \sqrt{1+\sin A}$ or $\sin\dfrac{A}{2}+\cos\dfrac{A}{2} = \pm\sqrt{1+\sin A}$

  i.e., $\begin{cases} +ve, \text{ if } 2n\pi - \dfrac{\pi}{2} \le -\dfrac{A}{2} \le 2n\pi + \dfrac{3\pi}{4} \\ -ve, \text{ otherwise} \end{cases}$
- $\left|\sin\dfrac{A}{2}-\cos\dfrac{A}{2}\right| = \sqrt{1-\sin A}$ or $\sin\dfrac{A}{2}-\cos\dfrac{A}{2} = \pm\sqrt{1-\sin A}$

  i.e., $\begin{cases} +ve, \text{ if } 2n\pi + \dfrac{\pi}{4} \le -\dfrac{A}{2} \le 2n\pi + \dfrac{5\pi}{4} \\ -ve, \text{ otherwise} \end{cases}$
- Maximum and Minimum values of $a\cos\theta + b\sin\theta$ are $\sqrt{a^2+b^2}$ and $-\sqrt{a^2+b^2}$

  i.e., $-\sqrt{a^2+b^2} \le a\cos\theta + b\sin\theta \le \sqrt{a^2+b^2}$
- $\sin^2 A - \sin^2 B = \sin(A+B)\sin(A-B)$

Previous Years' Questions

1. $\cos 1° \cdot \cos 2° \cdot \cos 3° \ldots \cos 179° =$

(a) 0 (b) 1 (c) $-\dfrac{1}{2}$ (d) -1 *(2018)*

2. If $2\sin\left(\theta+\dfrac{\pi}{3}\right) = \cos\left(\theta-\dfrac{\pi}{6}\right)$, then $\tan\theta =$

(a) $\sqrt{3}$ (b) $-\dfrac{1}{\sqrt{3}}$ (c) $\dfrac{1}{\sqrt{3}}$ (d) $-\sqrt{3}$

(2018)

3. If A, B, C are the angles of ΔABC, then $\cot A \cdot \cot B + \cot B \cdot \cot C + \cot C \cdot \cot A =$

(a) 0 (b) 1 (c) 2 (d) -1 *(2018)*

4. The value of $\sin 18°$ is

(a) $\dfrac{\sqrt{5}+1}{4}$ (b) $\dfrac{\sqrt{5}-1}{4}$ (c) $\dfrac{4}{\sqrt{5}+1}$ (d) $\dfrac{4}{\sqrt{5}-1}$

(2019)

5. If $\theta = \frac{17\pi}{3}$ then $\tan\theta - \cot\theta =$

(a) $\frac{1}{2\sqrt{3}}$ (b) $\frac{-1}{2\sqrt{3}}$

(c) $\frac{2}{\sqrt{3}}$ (d) $-\frac{2}{\sqrt{3}}$ *(2019)*

6. In ΔABC, if $\tan A + \tan B + \tan C = 6$ and $\tan A \cdot \tan B = 2$, then $\tan C =$

(a) 3 (b) 4

(c) 1 (d) 2 *(2019)*

7. If $A - B = \frac{\pi}{4}$, then $(1+\tan A)(1-\tan B) =$

(a) 4 (b) 3 (c) 1 (d) 2 *(2019)*

8. The value of $\cos^2 10° - \cos 10° \cdot \cos 50° + \cos^2 50°$ is

(a) $\frac{3}{2} + \cos 20°$ (b) $\frac{3}{4}$

(c) $\frac{3}{4}(1+\cos 20°)$ (d) $\frac{3}{2}$ *(2022)*

9. If $\sin(\theta - \alpha)$, $\sin\theta$ and $\sin(\theta + \alpha)$ are in H.P., then the value of $\cos 2\theta$ is

(a) $1 + 4\cos^2\frac{\alpha}{2}$ (b) $1 - 4\cos^2\frac{\alpha}{2}$

(c) $-1 - 4\cos^2\frac{\alpha}{2}$ (d) $-1 + 4\cos^2\frac{\alpha}{2}$ *(2023)*

10. If $a\cos 2\theta + b\sin 2\theta = c$ has α and β as its roots, then the value of $\tan\alpha + \tan\beta$ is

(a) $\frac{2b}{c+a}$ (b) $\frac{2a}{b+c}$ (c) $\frac{b}{c+a}$ (d) $\frac{a}{b+c}$ *(2023)*

Hints & Explanations

1. **(a) :** $\cos 1° \cdot \cos 2° \cdot \cos 3° \cdots \cos 179°$
$= \cos 1° \cdot \cos 2° \ldots \cos 90° \ldots \cos 179° = 0$

2. **(d) :** We have, $2\sin\left(\theta + \frac{\pi}{3}\right) = \cos\left(\theta - \frac{\pi}{6}\right)$

$$\Rightarrow 2\left(\sin\theta \times \frac{1}{2} + \cos\theta \times \frac{\sqrt{3}}{2}\right) = \cos\theta \times \frac{\sqrt{3}}{2} + \sin\theta \times \frac{1}{2}$$

$$\Rightarrow \frac{1}{2}\sin\theta + \frac{\sqrt{3}}{2}\cos\theta = 0 \Rightarrow \frac{1}{2}\sin\theta = \frac{-\sqrt{3}}{2}\cos\theta$$

$$\Rightarrow \tan\theta = -\sqrt{3}$$

3. **(b) :** $\because A + B + C = \pi \Rightarrow A + B = \pi - C$
Now, $\cot(A + B) = \cot(\pi - C)$

$$\Rightarrow \frac{\cot A \cot B - 1}{\cot A + \cot B} = -\cot C$$

$\Rightarrow \cot A \cot B + \cot B \cot C + \cot A \cot C = 1$

4. **(b) :** Let $x = 18° \Rightarrow 5x = 5 \times 18° = 90°$
$\Rightarrow 2x + 3x = 90° \Rightarrow 2x = 90° - 3x$
$\Rightarrow \sin 2x = \sin(90° - 3x) \Rightarrow \sin 2x = \cos 3x$
$\Rightarrow 2\sin x\cos x = 4\cos^3 x - 3\cos x$
$\Rightarrow 2\sin x\cos x = \cos x(4\cos^2 x - 3)$
$\Rightarrow 2\sin x = 4\cos^2 x - 3 \Rightarrow 2\sin x = 4(1 - \sin^2 x) - 3$
$\Rightarrow 2\sin x = 4 - 4\sin^2 x - 3$
$\Rightarrow 4\sin^2 x + 2\sin x - 1 = 0$
This is a quadratic in $\sin x$, where $a = 4$, $b = 2$, $c = -1$

$$\therefore \sin x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{4 - 4(4)(-1)}}{8}$$

$$= \frac{-2 \pm \sqrt{20}}{8} = \frac{-2 \pm 2\sqrt{5}}{8} = \frac{-1 \pm \sqrt{5}}{4}$$

But $\sin 18° > 0$, because $18°$ lies in first quadrant.

$$\therefore \sin 18° = \frac{\sqrt{5} - 1}{4}$$

5. **(d) :** Given, $\theta = \frac{17\pi}{3} = \frac{18\pi - \pi}{3} \Rightarrow \theta = 6\pi - \frac{\pi}{3}$

Now, $\tan\theta - \cot\theta = \tan\theta - \frac{1}{\tan\theta}$

$$= \tan\left(6\pi - \frac{\pi}{3}\right) - \frac{1}{\tan\left(6\pi - \frac{\pi}{3}\right)} = \tan\left(-\frac{\pi}{3}\right) - \frac{1}{\tan\left(-\frac{\pi}{3}\right)}$$

$$= -\sqrt{3} - \left(\frac{1}{-\sqrt{3}}\right) = \frac{-2}{\sqrt{3}}$$

6. **(a) :** We know that in ΔABC, $A + B + C = \pi$
$\Rightarrow A + B = \pi - C$
$\Rightarrow \tan(A + B) = \tan(\pi - C)$

$$\Rightarrow \frac{\tan A + \tan B}{1 - \tan A \tan B} = -\tan C \quad \ldots \text{(i)}$$

Now, $\tan A + \tan B + \tan C = 6$ (Given)
$\Rightarrow \tan A + \tan B = 6 - \tan C$
Substituting in (i), we get

$$\frac{6 - \tan C}{1 - 2} = -\tan C \quad [\because \tan A \cdot \tan B = 2 \text{ (given)}]$$

$\Rightarrow -(6 - \tan C) = -\tan C$

$\Rightarrow$ $\tan C - 6 = -\tan C$
$\Rightarrow$ $2\tan C = 6$
$\Rightarrow$ $\tan C = 3$

7. **(d) :** Given, $A - B = \frac{\pi}{4}$, then

$\tan(A-B) = \tan\frac{\pi}{4}$

$\frac{\tan A - \tan B}{1 + \tan A \tan B} = 1$

$\tan A - \tan B = 1 + \tan A \tan B$
$(1 + \tan A) - \tan B\,(1 + \tan A) = 2$
$(1 - \tan B)\,(1 + \tan A) = 2$

8. **(b) :** $\cos^2 10° - \cos 10° \cdot \cos 50° + \cos^2 50°$

$= \cos^2 50° + 1 - \sin^2 10° - \frac{1}{2}(2\cos 10° \cdot \cos 50°)$

$= \cos^2 50° - \sin^2 10° + 1 - \frac{1}{2}(\cos 60° + \cos 40°)$

$[\because 2\cos A \cos B = \cos(A+B) + \cos(A-B)]$

$= \cos 60° \cdot \cos 40° + 1 - \frac{1}{2}(\cos 60° + \cos 40°)$

$[\because \cos^2 A - \sin^2 B = \cos(A+B)\cos(A-B)]$

$= \frac{1}{2}\cos 40° + 1 - \frac{1}{2}\left(\frac{1}{2} + \cos 40°\right) = 1 - \frac{1}{4} = \frac{3}{4}$

9. **(b) :** $\sin(\theta - \alpha)$, $\sin\theta$ and $\sin(\theta + \alpha)$ are in H.P.

$\Rightarrow \frac{1}{\sin(\theta-\alpha)}, \frac{1}{\sin\theta}, \frac{1}{\sin(\theta+\alpha)}$ are in A.P.

$\Rightarrow \frac{1}{\sin(\theta-\alpha)} + \frac{1}{\sin(\theta+\alpha)} = \frac{2}{\sin\theta}$

$\Rightarrow \frac{\sin(\theta+\alpha) + \sin(\theta-\alpha)}{\sin(\theta-\alpha)\sin(\theta+\alpha)} = \frac{2}{\sin\theta}$

$\Rightarrow \sin\theta = \frac{2\sin(\theta-\alpha)\sin(\theta+\alpha)}{\sin(\theta+\alpha)+\sin(\theta-\alpha)}$

$\Rightarrow \sin\theta = \frac{2(\sin^2\theta - \sin^2\alpha)}{2\sin\theta\cos\alpha}$

$\Rightarrow \sin^2\theta\cos\alpha = \sin^2\theta - \sin^2\alpha \Rightarrow \sin^2\theta(1 - \cos\alpha) = \sin^2\alpha$
$\Rightarrow \sin^2\theta(1 - \cos\alpha) = (1 - \cos^2\alpha)$
$\Rightarrow \sin^2\theta(1 - \cos\alpha) = (1 - \cos\alpha)(1 + \cos\alpha)$
$\Rightarrow \sin^2\theta = 1 + \cos\alpha \Rightarrow 1 - \cos^2\theta = 1 + \cos\alpha$
$\Rightarrow \cos^2\theta = -\cos\alpha \Rightarrow 2\cos^2\theta = -2\cos\alpha$
$\Rightarrow 2\cos^2\theta - 1 = -2\cos\alpha - 1 \Rightarrow \cos 2\theta = -2\cos\alpha - 1$

$= -2\left(2\cos^2\frac{\alpha}{2} - 1\right) - 1 = -4\cos^2\frac{\alpha}{2} + 2 - 1 = 1 - 4\cos^2\frac{\alpha}{2}$

10. **(a) :** Given, $a\cos 2\theta + b\sin 2\theta = c$

$\Rightarrow a\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right) + b\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) = c$

$\Rightarrow a(1 - \tan^2\theta) + 2b\tan\theta = c(1 + \tan^2\theta)$

Since, α and β are roots of the given equation.

$\Rightarrow a(1 - \tan^2\alpha) + 2b\tan\alpha = c(1 + \tan^2\alpha)$ ...(i)

and $a(1 - \tan^2\beta) + 2b\tan\beta = c(1 + \tan^2\beta)$ ...(ii)

Subtracting (ii) from (i), we get

$-a(\tan^2\alpha - \tan^2\beta) + 2b(\tan\alpha - \tan\beta) = c(\tan^2\alpha - \tan^2\beta)$
$\Rightarrow 2b(\tan\alpha - \tan\beta) = (\tan^2\alpha - \tan^2\beta)(a + c)$
$\Rightarrow 2b = (\tan\alpha + \tan\beta)(a + c)$

$\Rightarrow \tan\alpha + \tan\beta = \frac{2b}{a+c}$

Solved Examples

Example 1 If $\tan A - \tan B = x$ and $\cot B - \cot A = y$, then find $\cot(A - B)$.

Soln.: We have, $\tan A - \tan B = x$

$\Rightarrow \frac{1}{\cot A} - \frac{1}{\cot B} = x \Rightarrow \frac{\cot B - \cot A}{\cot A \cot B} = x$

$\Rightarrow \cot A \cot B = \frac{y}{x}$

Now, $\cot(A-B) = \frac{\cot A \cdot \cot B + 1}{\cot B - \cot A}$

$= \frac{\frac{y}{x} + 1}{y} = \frac{x+y}{xy} = \frac{1}{x} + \frac{1}{y}$

Example 2 If A, B, C, D are angles of a cyclic quadrilateral, prove that $\cos A + \cos B + \cos C + \cos D = 0$.

Soln.: We know that the opposite angles of a cyclic quadrilateral are supplementary *i.e.*, $A + C = \pi$ and $B + D = \pi$.

$\therefore A = \pi - C$ and $B = \pi - D$

$\Rightarrow \cos A = \cos(\pi - C) = -\cos C$ and $\cos B$
$= \cos(\pi - D) = -\cos D$

$\therefore \cos A + \cos B + \cos C + \cos D$
$= -\cos C - \cos D + \cos C + \cos D = 0$

Example 3 If $\sin(\theta + \alpha) = a$, $\cos^2(\theta + \beta) = b$, then find the value of $\sin(\alpha - \beta)$.

Soln.: We have,

$\sin(\theta + \alpha) = a$ and $\cos(\theta + \beta) = \pm\sqrt{b}$

$\therefore \cos(\theta + \alpha) = \pm\sqrt{1 - a^2}$ and $\sin(\theta + \beta) = \pm\sqrt{1 - b}$

Now, $\sin(\alpha - \beta) = \sin\{(\theta + \alpha) - (\theta + \beta)\}$

$\Rightarrow \sin(\alpha-\beta) = \sin(\theta+\alpha)\cos(\theta+\beta) - \cos(\theta+\alpha)\sin(\theta+\beta)$

$\Rightarrow \sin(\alpha-\beta) = \pm a\sqrt{b} - \{\pm\sqrt{1-a^2}\}\times\{\pm\sqrt{1-b}\}$

$\Rightarrow \sin(\alpha-\beta) = \pm\{a\sqrt{b} - \sqrt{1-a^2}\sqrt{1-b}\}$

Example 4 If $3\tan\theta\tan\phi = 1$, Prove that $2\cos(\theta+\phi) = \cos(\theta-\phi)$.

Soln.: Given, $3\tan\theta\tan\phi = 1$ or $\cot\theta\cdot\cot\phi = 3$

or $\dfrac{\cos\theta\cos\phi}{\sin\theta\sin\phi} = \dfrac{3}{1}$

By componendo and dividendo, we have

$$\frac{\cos\theta\cos\phi+\sin\theta\sin\phi}{\cos\theta\cos\phi-\sin\theta\sin\phi} = \frac{3+1}{3-1} \Rightarrow \frac{\cos(\theta-\phi)}{\cos(\theta+\phi)} = 2$$

$\Rightarrow 2\cos(\theta+\phi) = \cos(\theta-\phi)$

Example 5 Show that $\tan\dfrac{\pi}{16} = \sqrt{4+2\sqrt{2}} - (\sqrt{2}+1)$.

Soln.: We know, $\tan\theta = \dfrac{1-\cos 2\theta}{\sin 2\theta}$ where $\theta = \dfrac{\pi}{16}$.

Also, $\sin 2\theta = \sqrt{\dfrac{1-\cos 4\theta}{2}} = \sqrt{\dfrac{1-\cos\frac{\pi}{4}}{2}} = \sqrt{\dfrac{1-\frac{1}{\sqrt{2}}}{2}} = \sqrt{\dfrac{\sqrt{2}-1}{2\sqrt{2}}}$

and $\cos 2\theta = \sqrt{\dfrac{1+\cos 4\theta}{2}} = \sqrt{\dfrac{1+\cos\frac{\pi}{4}}{2}} = \sqrt{\dfrac{1+\frac{1}{\sqrt{2}}}{2}} = \sqrt{\dfrac{\sqrt{2}+1}{2\sqrt{2}}}$

$$\therefore \tan\frac{\pi}{16} = \frac{1-\sqrt{\dfrac{\sqrt{2}+1}{2\sqrt{2}}}}{\sqrt{\dfrac{\sqrt{2}-1}{2\sqrt{2}}}} = \frac{\sqrt{2\sqrt{2}} - \sqrt{\sqrt{2}+1}}{\sqrt{\sqrt{2}-1}}$$

$$= \frac{\sqrt{\sqrt{2}+1}\left(\sqrt{2\sqrt{2}} - \sqrt{\sqrt{2}+1}\right)}{\sqrt{\sqrt{2}+1}\cdot\sqrt{\sqrt{2}-1}} = \frac{\sqrt{2\sqrt{2}(\sqrt{2}+1)} - \sqrt{(\sqrt{2}+1)^2}}{\sqrt{(\sqrt{2}+1)(\sqrt{2}-1)}}$$

$$= \sqrt{4+2\sqrt{2}} - (\sqrt{2}+1)$$

Example 6 If $\tan x + \tan(x+\pi/3) + \tan(x+2\pi/3) = 3$, then prove that $\tan 3x = 1$.

Soln.: The given equation can be written as

$$\tan x + \frac{\tan x + \tan(\pi/3)}{1-\tan x\tan(\pi/3)} + \frac{\tan x + \tan(2\pi/3)}{1-\tan x\tan(2\pi/3)} = 3$$

$$\Rightarrow \tan x + \frac{\tan x+\sqrt{3}}{1-\sqrt{3}\tan x} + \frac{\tan x-\sqrt{3}}{1+\sqrt{3}\tan x} = 3$$

$$\Rightarrow \tan x + \frac{(\tan x+\sqrt{3})(1+\sqrt{3}\tan x)+(1-\sqrt{3}\tan x)(\tan x-\sqrt{3})}{1-3\tan^2 x} = 3$$

$$\Rightarrow \tan x + \frac{8\tan x}{1-3\tan^2 x} = 3 \Rightarrow \frac{\tan x(1-3\tan^2 x)+8\tan x}{1-3\tan^2 x} = 3$$

$$\Rightarrow \frac{3(3\tan x - \tan^3 x)}{1-3\tan^2 x} = 3 \Rightarrow 3\tan 3x = 3 \Rightarrow \tan 3x = 1$$

Example 7 If $\cos^3\theta + \cos^3\left(\dfrac{2\pi}{3}+\theta\right) + \cos^3\left(\dfrac{4\pi}{3}+\theta\right) = a\cos 3\theta$, then find the value of a.

Soln.: $\cos\theta + \cos\left(\dfrac{2\pi}{3}+\theta\right) + \cos\left(\dfrac{4\pi}{3}+\theta\right)$

$= \cos\theta + \cos\left(\dfrac{2\pi}{3}+\theta\right) + \cos\left(\dfrac{2\pi}{3}-\theta\right)$

$[\because \cos\theta = \cos(2\pi-\theta)]$

$= \cos\theta + 2\cos\dfrac{2\pi}{3}\cos\theta = \cos\theta - \cos\theta = 0$

then, $\cos^3\theta + \cos^3\left(\dfrac{2\pi}{3}+\theta\right) + \cos^3\left(\dfrac{4\pi}{3}+\theta\right)$

$= 3\cos\theta\cos\left(\dfrac{2\pi}{3}+\theta\right)\cos\left(\dfrac{4\pi}{3}+\theta\right)$

Now, $a\cos 3\theta = \cos^3\theta + \cos^3\left(\dfrac{2\pi}{3}+\theta\right) + \cos^3\left(\dfrac{4\pi}{3}+\theta\right)$

$= 3\cos\theta\cos\left(\dfrac{2\pi}{3}+\theta\right)\cos\left(\dfrac{4\pi}{3}+\theta\right)$

$= 3\cos\theta\cos\left(\dfrac{2\pi}{3}+\theta\right)\cos\left(\dfrac{2\pi}{3}-\theta\right) = 3\cos\theta\left(\cos^2\theta - \sin^2\dfrac{2\pi}{3}\right)$

$= 3\cos\theta\left(\cos^2\theta - \dfrac{3}{4}\right) = \dfrac{3}{4}(4\cos^3\theta - 3\cos\theta) = \dfrac{3}{4}\cos 3\theta \Rightarrow a = \dfrac{3}{4}$

Example 8 Find the value of cos 12° + cos 84° + cos 156° + cos 132°.

Soln.: cos 12° + cos 84° + cos 156° + cos 132°

= (cos 12° + cos 132°) + (cos 84° + cos 156°)

$= 2\cos\left(\dfrac{12°+132°}{2}\right)\cos\left(\dfrac{132°-12°}{2}\right) + 2\cos\left(\dfrac{84°+156°}{2}\right)\cos\left(\dfrac{156°-84°}{2}\right)$

= 2 cos 72° cos 60° + 2 cos 120° cos 36°

= 2 sin 18° cos 60° + 2 cos 120° cos 36°

$= 2\left(\dfrac{\sqrt{5}-1}{4}\right)\dfrac{1}{2} + 2\left(\dfrac{\sqrt{5}+1}{4}\right)\left(-\dfrac{1}{2}\right) = -\dfrac{1}{2}$

Example 9 If $\sin\theta = \dfrac{-4}{5}$ and θ lies in third quadrant, then find the value of $\cos\dfrac{\theta}{2}$.

Soln.: We have given, $\sin\theta = -\dfrac{4}{5}$

$\Rightarrow \quad \cos\theta = \pm\sqrt{1-\sin^2\theta} = \pm\sqrt{1-\dfrac{16}{25}} = \pm\sqrt{\dfrac{25-16}{25}} = \pm\dfrac{3}{5}$

$\Rightarrow \quad \cos\theta = -\dfrac{3}{5}$ ($\because$ $\cos\theta < 0$ in III[rd] quadrant)

$\Rightarrow \quad 2\cos^2\frac{\theta}{2} - 1 = \frac{-3}{5}$ $\quad (\because \ \cos 2\theta = 2\cos^2\theta - 1)$

$\Rightarrow \quad 2\cos^2\frac{\theta}{2} = 1 - \frac{3}{5}$

$\Rightarrow \quad 2\cos^2\frac{\theta}{2} = \frac{2}{5}$

$\Rightarrow \quad \cos\frac{\theta}{2} = \pm\frac{1}{\sqrt{5}}$

$\Rightarrow \quad \cos\frac{\theta}{2} = -\frac{1}{\sqrt{5}}$

[$\because$ If θ lies in IIIrd quadrant then θ/2 lies in IInd quadrant]

Example 10 Let α and β be two real roots of the equation $(k+1)\tan^2 x - \sqrt{2}\cdot\lambda\tan x = (1-k)$, where $k(\neq -1)$ and λ are real numbers. If $\tan^2(\alpha+\beta) = 50$, then find the value of λ.

Soln.: Given, $(k + 1)\tan^2 x - \sqrt{2}\lambda \tan x + (k - 1) = 0$

$\therefore \quad \tan\alpha + \tan\beta = \frac{\sqrt{2}\lambda}{k+1}$ and $\tan\alpha\cdot\tan\beta = \frac{k-1}{k+1}$

Now, $\tan(\alpha+\beta) = \frac{\frac{\sqrt{2}\lambda}{k+1}}{1-\frac{k-1}{k+1}} = \frac{\sqrt{2}\lambda}{2} = \frac{\lambda}{\sqrt{2}}$

$\Rightarrow \quad \tan^2(\alpha+\beta) = \frac{\lambda^2}{2} = 50$ [Given]

$\Rightarrow \quad \lambda = \pm 10$

Daily Practice Problems

Trigonometric Functions of Sum and Difference of Angles

1. The value of $\sqrt{3}$ cosec 20° – sec 20° is

(a) 1 (b) 7 (c) 2 (d) 4

2. The value of cos (35° + *A*) cos (35° – *B*) + sin (35° + *A*) sin (35° – *B*) is equal to

(a) sin (*A* + *B*) (b) sin (*A* – *B*)

(c) cos (*A* + *B*) (d) cos (*A* – *B*)

3. The value of sin(45° + θ) – cos(45° – θ) is

(a) 2cosθ (b) 2sinθ (c) 1 (d) 0

4. $\sin\left(\theta - \frac{\pi}{6}\right) + \cos\left(\theta - \frac{\pi}{3}\right)$ is equal to

(a) $\sqrt{3}\cos\theta$ (b) $\sqrt{3}\sin\theta$ (c) $-\sqrt{3}\sin\theta$ (d) $-\sqrt{3}\cos\theta$

5. The value of $\sqrt{2}\sin\left(\frac{\pi}{4} - \theta\right)$ is

(a) –(sin θ + cos θ) (b) sin θ – cos θ

(c) cos θ + sin θ (d) cos θ – sin θ

6. If sin α + sin β = $\frac{\sqrt{6}}{2}$ and cos α + cos β = $\frac{\sqrt{2}}{2}$, then cos(α – β) is equal to

(a) $\frac{1}{2}$ (b) $\frac{3}{2}$ (c) $\frac{-1}{2}$ (d) 0

7. If $2\sin\left(\theta + \frac{\pi}{3}\right) = \cos\left(\theta - \frac{\pi}{6}\right)$, then $\tan\theta + \sqrt{3} =$

(a) 1 (b) –1 (c) 0 (d) ∞

8. Find the value of

sin(40° + θ) cos(10° + θ) – cos(40° + θ) sin(10° + θ) .

(a) 1/2 (b) 1/4 (c) 1 (d) 0

9. The value of sin 70° cos 10° – cos 70° sin 10° is equal to

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{\sqrt{3}}{4}$ (c) $\frac{\sqrt{3}}{8}$ (d) $\frac{\sqrt{3}}{16}$

10. The value of cos 15° is equal to

(a) $\frac{\sqrt{3}-1}{2\sqrt{2}}$ (b) $\frac{\sqrt{3}+1}{2\sqrt{2}}$ (c) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$ (d) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$

11. If tan*A* and tan*B* are the roots of the quadratic equation, $3x^2 - 10x - 25 = 0$, then the value of $3\sin^2(A + B) - 10 \sin(A + B)\cdot\cos(A + B) - 25\cos^2(A + B)$ is

(a) 10 (b) –10

(c) 25 (d) –25

Trigonometric Functions of Allied Angles

12. The value of tan1° tan2° tan3° ... tan89° is

(a) 0 (b) 1

(c) 1/2 (d) Not defined

13. The value of $\frac{\tan 330° \sec 420° \sin 300°}{\tan 135° \sin 210° \sec 315°}$ is equal to

(a) $\frac{1}{\sqrt{2}}$ (b) $\sqrt{2}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\sqrt{3}$

14. The value of $\tan\frac{19\pi}{3}$ is

(a) $\frac{1}{\sqrt{3}}$ (b) $\sqrt{3}$

(c) $-\sqrt{3}$ (d) None of these

15. The value of $\sin\left(-\frac{11\pi}{3}\right)$ is

(a) $\sqrt{3}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{2}{\sqrt{3}}$ (d) $\frac{1}{\sqrt{3}}$

16. The value of tan(1°) + tan(89°) is

(a) $\frac{1}{\sin(1°)}$ (b) $\frac{1}{\sin(2°)}$

(c) $\frac{1}{\sin(1°)\cos(1°)}$ (d) $\frac{2}{\sin(1°)}$

17. The value of tan 50° is

(a) tan 10° + 2 tan 40° (b) tan 40° + 2 tan 10°

(c) tan 10° – 2 tan 40° (d) tan 10° – 2 tan 10°

18. $\frac{\sin(90°+A)}{\cos(-A)}-\frac{\sin(180°-A)}{\sin(-A)}+\frac{\tan(270°+A)}{\cot(-A)}=$

(a) – 1 (b) 3 (c) –3 (d) 1

19. The value of $\cos\frac{\pi}{12}+\cos\frac{3\pi}{12}+\cos\frac{9\pi}{12}+\cos\frac{11\pi}{12}$ is equal to

(a) 0 (b) 1

(c) –1 (d) None of these

20. The value of $\frac{\cos(90°+\theta)\sec(270°+\theta)\sin(180°+\theta)}{\cos(-\theta)\cos(270°-\theta)\tan(180°+\theta)}$ is equal to

(a) $\text{cosec}\,\theta$ (b) $\sec\theta$

(c) $\sin\theta$ (d) $-\text{cosec}\,\theta$

Trigonometric Functions of Multiple Angles

21. $\sqrt{2+\sqrt{2+\sqrt{2+2\cos 8\theta}}}=$

(a) sin 2θ (b) 2 sinθ

(c) 2 cosθ (d) $2\cos\frac{\theta}{2}$

22. Let $f_k(x)=\frac{1}{k}(\sin^k x+\cos^k x)$ for k = 1, 2, 3, ... Then for all $x \in R$, the value of $f_4(x) - f_6(x)$ is equal to

(a) $\frac{5}{12}$ (b) $\frac{1}{4}$ (c) $\frac{-1}{12}$ (d) $\frac{1}{12}$

23. If $(1+\sqrt{1+x})\tan x=1+\sqrt{1-x}$, then sin 4x is equal to

(a) 4x (b) 2x

(c) x (d) None of these

24. If 'θ' is in the III quadrant, then

$\sqrt{4\sin^4\theta+\sin^2 2\theta}+4\cos^2\left(\frac{\pi}{4}-\frac{\theta}{2}\right)=$

(a) 2 (b) –2 (c) 0 (d) 1

25. If $\cos x=\frac{2\cos y-1}{2-\cos y}$, where $x, y \in (0, \pi)$, then $\tan\frac{x}{2}\cdot\cot\frac{y}{2}$ is equal to

(a) $\sqrt{2}$ (b) $\sqrt{3}$ (c) $\frac{1}{\sqrt{2}}$ (d) $\frac{1}{\sqrt{3}}$

26. If $a^2 - 2a\cos x + 1 = 674$ and tan (x/2) = 7, then the integral value of a is

(a) 25 (b) 49

(c) 67 (d) 74

27. If sin x + cos x + tan x + cot x + sec x + cosec x = 7 and $\sin 2x = a-b\sqrt{7}$, then ordered pair (a, b) can be

(a) (6, 2) (b) (8, 3) (c) (22, 8) (d) (11, 4)

28. If $u = \sin^6 x + \cos^6 x$ then maximum and minimum values respectively of u are

(a) $\frac{1}{4}$ and $\frac{-1}{4}$ (b) 1 and –1

(c) 1 and $\frac{1}{4}$ (d) $\frac{1}{4}$ and –1

29. If $\text{cosec}\,\theta=\left[\frac{\tan^2[(\alpha-\pi)/4]-1}{\tan^2[(\alpha-\pi)/4]+1}+\cos\frac{\alpha}{2}\cot 4\alpha\right]\sec\frac{9\alpha}{2}$, then θ =

(a) α (b) 2α (c) π/2 – 2α (d) 4α

30. If $\cos 5\theta = a_0 + a_1\cos\theta + a_2\cos^2\theta + a_3\cos^3\theta + a_4\cos^4\theta + a_5\cos^5\theta$, then the value of $\sum_{i=0}^{5}a_i$ is equal to

(a) 0 (b) 1 (c) 2 (d) 3

31. $\tan\frac{\pi}{5}+2\tan\frac{2\pi}{5}+4\cot\frac{4\pi}{5}=$

(a) $\cot\frac{\pi}{5}$ (b) $\cot\frac{2\pi}{5}$ (c) $\cot\frac{3\pi}{5}$ (d) $\cot\frac{4\pi}{5}$

32. If $5(\tan^2 x - \cos^2 x) = 2\cos 2x + 9$, then the value of cos4x is

(a) $\frac{1}{3}$ (b) $\frac{2}{9}$ (c) $-\frac{7}{9}$ (d) $-\frac{3}{5}$

Factorization Formulae

33. If $\cos A=\frac{3}{4}$, then find the value of $32\sin\frac{A}{2}\sin\frac{5A}{2}$.

(a) 11 (b) 10 (c) 8 (d) 2

34. If x and y are acute angles such that $\cos x + \cos y = \frac{3}{2}$ and $\sin x + \sin y = \frac{3}{4}$, then sin (x + y) =

(a) $\frac{3}{4}$ (b) $\frac{3}{5}$ (c) $\frac{4}{5}$ (d) $\frac{2}{5}$

35. If $\cos^3 x\sin 2x=\sum_{m=1}^{n}a_m\sin mx$ is an identity in x, then which one of the following is incorrect?

(a) $a_2=0,\ a_3=\frac{3}{8}$ (b) $a_1=\frac{1}{2},\ n=6$

(c) $a_1=\frac{1}{4},\ n=5$ (d) $\sum a_m=\frac{3}{4}$

36. The value of $\frac{\left(\begin{array}{l}\cos 5°\cos 20°+\cos 35°\cos 50°\\-\sin 5°\sin 20°-\sin 35°\sin 50°\end{array}\right)}{\left(\begin{array}{l}\sin 5°\cos 20°-\sin 35°\cos 50°\\+\cos 5°\sin 20°-\cos 35°\sin 50°\end{array}\right)}$ is

(a) $-\frac{1}{\sqrt{3}}$ (b) $\frac{1}{\sqrt{3}}$ (c) $-\sqrt{3}$ (d) $\sqrt{3}$

37. The value of sin 12° sin 48° sin 54° is
(a) 1/2 (b) 1/4 (c) 1/8 (d) 1/16

38. Let $\frac{\sin(\theta-\alpha)}{\sin(\theta-\beta)}=\frac{a}{b}, \frac{\cos(\theta-\alpha)}{\cos(\theta-\beta)}=\frac{c}{d}$. Then the value of $\cos(\alpha-\beta)$ equals

(a) $\frac{ac-bd}{ad+bc}$ (b) $\frac{ac+bd}{ad+bc}$

(c) $\frac{ac+bd}{ab+cd}$ (d) $\frac{ac-bd}{ab+cd}$

39. If $-\pi \le x \le \pi$, $-\pi \le y \le \pi$ and $\cos x + \cos y = 2$, then the value of $\cos(x-y)$ is
(a) −1 (b) 0
(c) 1 (d) none of these

40. If $\sin(\theta+\phi) = n\sin(\theta-\phi)$, $n \ne 1$, then the value of $\frac{\tan\theta}{\tan\phi}$ is equal to

(a) $\frac{n}{n-1}$ (b) $\frac{n+1}{n-1}$ (c) $\frac{n}{1-n}$ (d) $\frac{n-1}{n+1}$

Trigonometric Functions of Angles of a Triangle

41. In a ΔABC, cosec $A(\sin B\cos C + \cos B\sin C) =$
(a) 0 (b) 1
(c) −1 (d) None of these

42. In a ΔABC, $\cos^2 A + \cos^2 B + \cos^2 C =$

(a) $1-2\cos A\cos\frac{B}{2}\cos\frac{C}{2}$ (b) $1-2\cos\frac{A}{2}\cos\frac{B}{2}\cos C$

(c) $1-2\cos\frac{A}{2}\cos B\cos\frac{C}{2}$ (d) $1-2\cos A\cos B\cos C$

43. If $A+B+C=\pi$, then the value of $\sin A + \sin B - \sin C =$

(a) $\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$ (b) $2\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

(c) $3\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$ (d) $4\sin\frac{A}{2}\sin\frac{B}{2}\cos\frac{C}{2}$

44. In a triangle ABC with $\angle C = 90°$, the equation whose roots are $\tan A$ and $\tan B$ is

(a) $x^2-\left(\frac{2}{\sin A}\right)x+1=0$

(b) $x^2-(2\sin 2A)\,x+1=0$

(c) $x^2-(2\sin 2A)\,x+1=0$

(d) $x^2-\frac{x}{\sin 2A}+1=0$

45. Given angle C of a triangle ABC to be obtuse and $\sin(A+B)=\frac{\sqrt{3}}{2}$ and $\cos(A-B)=\frac{1}{\sqrt{2}}$, then C
(a) 160° (b) 110°
(c) 120° (d) 150°

46. Find the angles of a triangle, given that angle A is obtuse and sec $(B+C)$ = cosec $(B-C)$ = 2
(a) 120°, 45°, 15° (b) 120°, 30°, 30°
(c) 135°, 15°, 40° (d) 90°, 45°, 45°

47. In a ΔABC, if $\sin(A) = 5/13$ and $\sin(B) = 99/101$, then the value of $(1313\cos(C))$ is
(a) 255 (b) 265
(c) 275 (d) 770

48. In any triangle ABC, $\cos(A+B) + \cos C =$
(a) 1 (b) −1
(c) 2 (d) 0

49. In any triangle ABC, $\cos\frac{A+B}{2}$
(a) sin C (b) tan C/2
(c) cos C/4 (d) sin C/2

50. In a ΔABC, $\tan A$ and $\tan B$ are roots of $pq(x^2+1) = r^2x$. Then ΔABC is
(a) a right angled triangle
(b) an acute angled triangle
(c) an obtuse angled triangle
(d) an equilateral triangle

ANSWER KEYS

1. (d)	**2.** (c)	**3.** (d)	**4.** (b)	**5.** (d)	**6.** (d)	**7.** (c)	**8.** (a)	**9.** (a)	**10.** (b)
11. (d)	**12.** (b)	**13.** (b)	**14.** (b)	**15.** (b)	**16.** (c)	**17.** (b)	**18.** (b)	**19.** (a)	**20.** (d)
21. (c)	**22.** (d)	**23.** (c)	**24.** (a)	**25.** (b)	**26.** (a)	**27.** (c)	**28.** (c)	**29.** (d)	**30.** (b)
31. (a)	**32.** (c)	**33.** (a)	**34.** (c)	**35.** (b)	**36.** (c)	**37.** (c)	**38.** (b)	**39.** (c)	**40.** (b)
41. (b)	**42.** (d)	**43.** (d)	**44.** (a)	**45.** (c)	**46.** (a)	**47.** (a)	**48.** (d)	**49.** (d)	**50.** (a)

BIOLOGY

CHAPTER

1

Biomolecules

Important Facts

- ***Definition***
  - ***Biochemistry*** *is the branch of science that provides us the idea of the chemistry of living organisms and molecular basis for changes taking place in plants, animals and microbial cells.*
- ***Biomolecules in the cell***
  - ***Carbohydrates*** *are most abundant biomolecules on earth. These are composed of carbon, hydrogen and oxygen. The general formula is* $C_n(H_2O)_n$ *or* $(CH_2O)_n$*. Living organisms use carbohydrates as accessible energy to fuel cellular reactions.*
  - *All monosaccharides are reducing sugars due to presence of free aldehyde or ketone group. These sugars reduce the* ***Benedict's reagent*** *(*Cu^{2+} *to* Cu^{+}*) since they are capable of transferring hydrogens (electrons) to other compounds, a process called reduction.*
  - ***Lipids*** *are made up of glycerol and fatty acids. Glycerol is made up of three carbon atoms with a hydroxyl group attached to it and fatty acids consists of an acid group at one end of a hydrocarbon chain. Fatty acid may be* ***saturated*** *(no double bond) or* ***unsaturated*** *(one or more double bonds).*
  - *The term* ***'protein'*** *was suggested by Berzelius (1830). Mulder adopted the term protein to refer to the complex, organic nitrogenous substances found in the cell of all animals and plants.*
  - *Proteins are the most diverse biomolecules on earth. These are variously folded linear polymers of amino acids. Linear chains of amino acids are linked by peptide bonds and are called polypeptides. Proteins perform various structural and physiological functions in living beings.*
  - ***Rennet*** *tablets which are used for coagulating milk protein, casein (cheese) contain rennin enzyme that is obtained from the stomach of calf.*
  - ***Nucleic acid*** *is formed by end-to-end polymerisation of large number of repeated units called nucleotides. A nucleotide contains - a* ***nitrogen base****, a* ***pentose sugar*** *and* ***phosphate group****. The function of nucleic acid is to store and express genetic information.*
  - *Certain organisms like bacteriophage* $\phi \times 174$ *and several bacterial viruses possess single stranded DNA.*

Important Notes

Introduction

- Elemental analysis gives elemental composition of living tissues in the form of hydrogen, oxygen, chlorine, carbon, etc., while analysis for compounds gives an idea of the kind of organic and inorganic constituents present in living tissues.
- All living organisms have basic three types of macromolecules, which are polymers of simple subunits called **monomers**. For example, polysaccharides (carbohydrates), polypeptides (proteins) and polynucleotides (nucleic acids) are the polymers of monosaccharides, amino acids and nucleotides, respectively.
- Lipids are water insoluble and small molecular weight compounds as compared to macromolecules.
- Inorganic biomolecules can be prime elements (C, H and O), macroelements (N, P, K, Ca, Mg, S) and trace elements (B, Cl, Zn, Cu, Fe, Na, Mo, Ni, Si, Co).

Biomolecules in the Cell

A. Carbohydrates

- The term carbohydrates mean 'hydrates of carbon'. They are composed of carbon, hydrogen and oxygen and have a general formula $(CH_2O)_n$, where 'n' is an integer. They contain hydrogen and oxygen in the same ratio as in water (2:1).

1. Monosaccharides

- They are simplest carbohydrate monomers which cannot be hydrolysed into smaller components and are the building blocks or monomers of complex carbohydrates.
- Monosaccharides that contain an aldehyde (–CHO) group are referred to as **aldoses**, *e.g.*, glucose, xylose,

etc., and those with a ketone (–C=O) group are referred to as **ketoses**, *e.g.*, ribulose, fructose, etc.

Table : Different types of monosaccharides

Number of carbon atoms	General term	Examples
3	Triose	Glyceraldehyde
4	Tetrose	Erythrose
5	Pentose	Ribose (RNA) and Deoxyribose (DNA)
6	Hexose	Glucose, fructose
7	Heptose	Sedoheptulose

- Examples of monosaccharides:
  (a) **Glucose** is the most important fuel in living cells. Energy is released when the glucose is metabolised during cellular respiration. In human blood, its concentration is about 90 mg per 100 mL of blood.
  Glucose molecules easily pass through the cell membrane into the cell due to its small size and solubility.

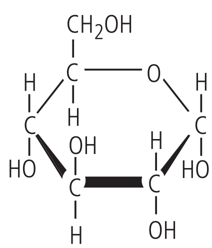


Fig.: Glucose

(b) **Galactose** is similar to glucose in its structure differing only in the position of one hydroxyl group. Galactose cannot play the same role in respiration as glucose.
(c) **Fructose** is the fruit sugar and chemically it is ketohexose.

2. **Disaccharides**

- They are formed by the condensation of two molecules of monosaccharides.
- In disaccharide, monosaccharide units (similar or dissimilar) are held together by a **glycosidic bond**. Glycosidic bond is formed by dehydration between two carbon atoms of two adjacent monosaccharides resulting in production of a molecule of water.
- Disaccharides are soluble in water, but they are too big to pass through the cell membrane by diffusion. They are broken down into monosaccharides in the small intestine during digestion which are then pass into the blood and through cell membranes into the cells.
- Examples of disaccharides:
  (a) **Sucrose:** It is formed by condensation of one molecule each of **glucose** and **fructose**. Sucrose is a **non-reducing sugar** as it lacks free aldehyde or ketone group.
  (b) **Lactose:** It is a **reducing sugar** which is formed by **condensation** of β-**glucose** and β-**galactose**.
  (c) **Maltose:** It is a reducing sugar which is formed by condensation of two molecules of glucose.

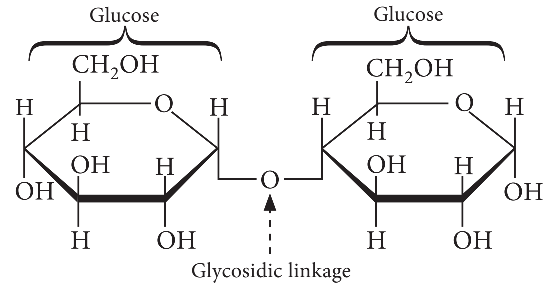


Fig.: Maltose

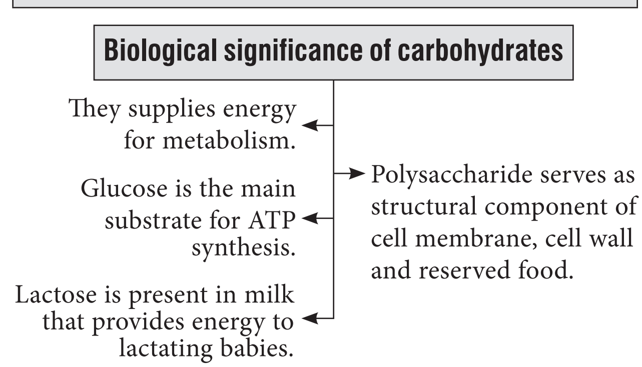
3. **Polysaccharides**

- Polysaccharides consist of repeating units of monosaccharides which have undergone a series of condensation reactions, adding one unit after the other to the chain till a very large molecule is formed. This is called **polymerisation**. Their properties depend on its length, branching, folding and coiling.
- **Homopolysaccharides :** Polymer of one type of monosaccharides.
- Examples :

Starch	It exists in two forms namely, amylose (unbranched) and amylopectin (branched, insoluble) which are made of α-glucose. It is stored as food in the plants.
Glycogen	It is amylopectin with very short distances between the branching side-chains. It is mainly stored in liver and muscles of animals.
Cellulose	It is a straight chain, polymer of β-glucose. It is the main constituent of plant cell wall.

- **Heteropolysaccharides :** Polymer of different types of monosaccharides, *e.g.*, hyaluronic acid, heparin, blood group substances, chondroitin sulphate.

Exoskeletons of arthropods, for example, have a complex polysaccharide called chitin. These complex polysaccharides are mostly homopolymers.



B. Lipids

- Lipids are biomolecules with greasy consistency that with long hydrocarbon chain containing C, H and O in which the content of oxygen is always small as compared to hydrogen(Hydrogen : Oxygen : : 2 : 1).

- Lipid is a broader term used for fatty acids and their derivatives. They are soluble in organic (non-polar) solvents like ether, benzene, etc.

Types of fatty acids

- **Saturated fatty acids**
  They are straight chain, fatty acids with no double bonds between C atoms of the hydrocarbon *E.g.*, palmitic acid and stearic acid.
- **Unsaturated fatty acids**
  They are with one or more double bonds between C atoms of the hydrocarbon chain. *E.g.*, oleic acid and linoleic acid.

- **Classification of lipids**
  (a) **Simple lipids :** These are esters of fatty acids with various alcohols. Examples : fats and wax.
  (i) **Fats:** They are esters of fatty acids with glycerol. Generally, unsaturated fats are liquid at room temperature and are called oils. These are hydrogenated to produce fats, *e.g.*, vanaspati ghee.

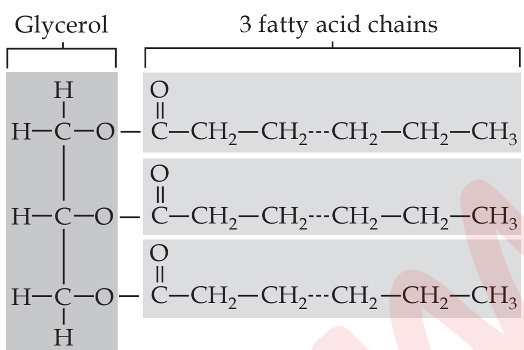


Fig.: Triglyceride

In animals, fat is stored in the adipocytes of the adipose tissue. In plants fat is stored in seeds to nourish embryo during germination. Fats deposited in subcutaneous tissue act as an insulator and reduce the body heat loss. Fats deposited around the internal organs act as cushions to absorb mechanical shocks.
(ii) **Wax:** They are esters of long chain fatty acids with long chain alcohols. They are solid at room temperature. They are most abundant in the blood, the gonads and the sebaceous glands of the skin. Waxes form water insoluble coating on hair and skin in animals and an outer coating on stems, leaves and fruits.
(b) **Compound lipids :** These are esters of fatty acids containing groups like phosphate, sugar, etc.
(i) Phospholipids have a nonpolar hydrophobic tail consisting of fatty acid chains and a polar hydrophilic head comprising a negatively charged phosphate and nitrogenous group. They are found in cell membrane. Example: Lecithin
(ii) Glycolipids contain glycerol, fatty acids, simple sugars such as galactose and nitrogenous base. They are also called **cerebrosides**. Large amounts of them have been found in the brain white matter and myelin sheath.
(c) **Sterols :** They are derived lipids which consists of fused hydrocarbon rings (steroid nucleus) and a long hydrocarbon side chain.
(i) The most abundant steroid in the nervous tissues is **cholesterol**. Adrenocorticoids, progesterone, testosterone, vitamin D, etc., are synthesised from cholesterol.
(ii) In plants, sterols exist chiefly as **phytosterols**. Yam Plant (*Dioscorea*) produces a steroid compound called diosgenin. It is used in the manufacture of antifertility pills, *i.e.*, birth control pills.

C. Proteins

- Proteins are polypeptides formed from linear chains of amino acids linked together by peptide bond. In a protein molecule, the amino acid units are linked together by peptide bonds (—NH—CO) formed between the amino and the carboxyl groups of successive amino acids.
- The four-levels of structural organisation in proteins are:
  (i) **Primary structure:** It is the linear sequence of amino acids in a polypeptide chain. Primary structure describes the basic structure of a protein, *i.e.*, number and sequence of amino acids in each polypeptide constituting the protein.
  (ii) **Secondary structure:** It is the development of new stearic relationships of amino acids present in the linear sequence inside the polypeptides. There are two types of secondary structures. (1) α **helix** - Right handed rotation of spirally coiled chain that are held together by intramolecular hydrogen bonding, *e.g.*, keratin. (2) β **helix** (β-pleated sheet) - Left-handed rotation of spirals that are held together by intermolecular hydrogen bond. *E.g.*, Protein of silk fibres.
  (iii) **Tertiary structure :** It is formed by the further complex bending and folding of secondary structure to form various types like sphere, rod or fibre. It is formed by disulphide bonds. *E.g.*, myoglobin.
  (iv) **Quaternary structure:** It consists of two or more polypeptide chains. Each polypeptide develops its own tertiary structure and functions as subunit of protein. *E.g.*, haemoglobin, etc.
- Proteins are extremely reactive and highly specific in behaviour. Proteins are **amphoteric** in nature, *i.e.*, they act as both acids and bases. The behaviour of proteins is strongly influenced by pH.
- Like amino acids, proteins are dipolar ions at the **isoelectric point**, *i.e.*, the sum of the positive charges is equal to the sum of the negative charges and the net charge is zero. The ionic groups of a protein are contributed by the side chains of the polyvalent amino acids.
- **Basic proteins:** It exists as cation and consists of more basic amino acids (lysine, arginine). For example, histones.
  Acidic proteins: It exists as anion and are rich in acidic amino acids. For example, most of the blood proteins.

Classification of proteins

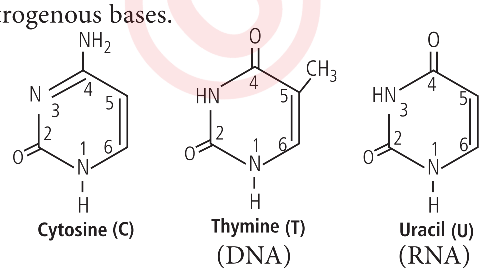
- **Simple proteins:** They yield amino acid residues upon hydrolysis. They may be soluble in water. For example, histones, albumins. Globular molecules of histones are not coagulated by heat. Albumins are also soluble in water but they get coagulated on heating.
- **Conjugated proteins**: They consist of a simple protein united with some non-protein group called prosthetic group, *e.g.*, haemoglobin.
  Conjugated proteins can be classified as:
  (i) Mucoproteins are carbohydrate-protein complexes, *e.g.*, mucin of saliva and heparin of blood.
  (ii) Lipoproteins are lipid-protein complexes, *e.g.* , conjugate protein found in brain, plasma membrane, milk, etc.
- **Derived proteins:** They are derived from native proteins molecules on hydrolysis, *e.g.*, metaproteins, peptones.

D. Nucleic Acids

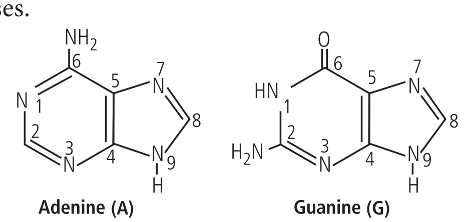
- The nucleic acids are among the largest of all molecules found in living beings. For nucleic acids, the building block (monomer) is a nucleotide. A nucleotide has three chemically distinct components:
  (i) A pentose sugar
  (ii) Phosphoric acid
  (iii) Nitrogen containing organic bases

Types of bases

- **Pyrimidines:** They are single ring (monocyclic) nitrogenous bases.



- **Purines:** They are double ring (dicyclic) nitrogenous bases.



Structure of DNA

- DNA is polymer of **deoxyribonucleotides.** It is the genetic material of all organisms with the exception of riboviruses.
- DNA is a long chain macromolecule made up of alternate sugar and phosphate groups which are formed by end-to-end polymerisation of large number of repeated units called nucleotides.
- The molecule consisting of nitrogenous base and sugar is termed **nucleoside**. Thus, **nucleotides** are made up of **nucleosides** and **phosphate**.
- In a nucleoside, nitrogenous base is attached to the first carbon atom (C-1) of the sugar and when a phosphate group gets attached with that of the carbon (C-5) atom of the sugar molecule a nucleotide molecule is formed.
- A single strand of DNA consists of several thousands of nucleotides one above the other with the phosphate group of the lower nucleotide attached with the 5′C of the deoxyribose sugar forms phosphodiester bond with that of the, 3′C of the deoxyribose sugar of the above nucleotide.
- Watson and Crick proposed **double helix model** for the structure of DNA which is depicted in the following figure:

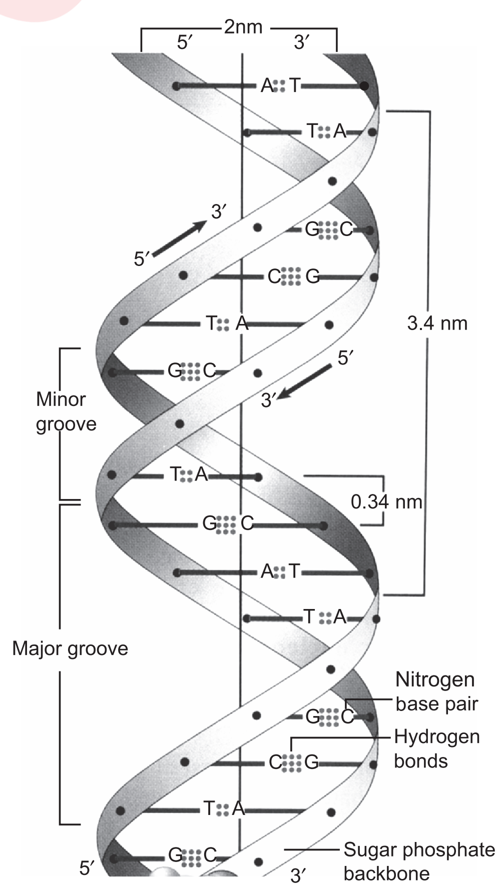


Fig.: Watson-Crick Double Helical DNA Molecule

- In 1950, Erwin Chargaff estimated the relative amounts of nitrogenous bases in DNA and his observations can be summarised as follows:
  (i) The purines and pyrimidines are always in equal amounts, *i.e.*, **A + G = T + C**.
  (ii) The amount of adenine is always equal to that of thymine, and the amount of guanine is always equal to that of cytosine, *i.e.*, **A = T** and **G = C**.
  (iii) The base ratio **A + T / G + C** may vary from one species to another, but remains constant for a particular species.

Ribonucleic acid (RNA)

- RNA is hereditary material in certain organisms like tobacco mosaic virus.
- Like DNA, RNA also consists of polynucleotide chain with the difference that it consists of single strand. In some cases, *e.g.*, reovirus and wound tumour virus, RNA is double stranded.
- The nucleotides of RNA have ribose sugar.

Types of RNA

Messenger RNA (*m*RNA)
It is a linear polynucleotide and constitutes 3% of cellular RNA.
*m*RNA molecule carry information to form a complete polypeptide chain called cistron.

Ribosomal RNA (*r*RNA)
It forms 50-60% part of ribosomes. It is synthesised in nucleus and constitutes 80-90% of cellular RNA. It gets coiled here and there due to intrachain complementary base pairing.

Transfer or soluble RNA (*t*RNA)
It constitutes 15% of total RNA and also called as soluble RNA. It consists of 70-80 nucleotides.
It is also single stranded but to number of complementary base sequences after pairing, it is shaped like clover-leaf.

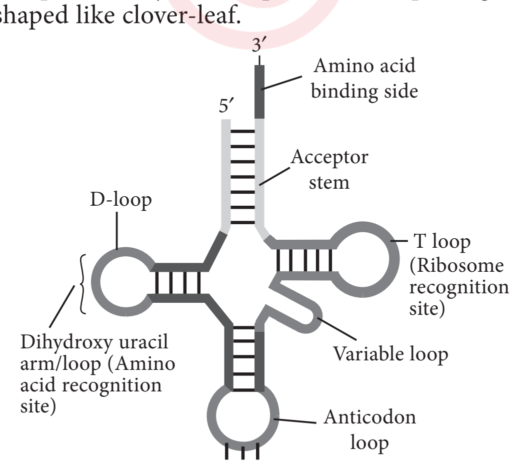


Fig.: The *t*RNA

E. Enzymes

- Enzymes are organic catalysts which catalyse biochemical reactions without being utilised themselves. All enzymes have specific 3-D conformation. German chemist Edward Buchner discovered enzymes by accident. Buchner discovered that living cells were not necessary but that yeast extract could bring about fermentation.
- Enzymes can be classified as:
  (i) **Exoenzymes :** Functional outside living cells, *e.g.*, enzymes released by many fungi.
  (ii) **Endoenzymes :** Functional inside living cells, *e.g.*, enzymes produced in mitochondria and chloroplast.

Nature of Enzymes

Purely proteinaceous enzymes
They are made up of protein only. *E.g.*, proteases like pepsin, trypsin, etc.

Conjugated enzymes
They are made up of two parts, *i.e.*, apoenzyme (protein part) and prosthetic group (non-protein part). The complete enzyme is called holoenzyme.

- The prosthetic group is firmly bound to the protein component by chemical bonds and is not removed by hydrolysis.
- The organic compounds that are tightly attached to the protein part are called co-enzymes. *E.g.*, nicotinamide-adenine dinucleotide (NAD) and flavin mononucleotide (FMN).
- The inorganic ions which are loosely attached to the protein part are called co-factors. *E.g.*, magnesium, copper, zinc, iron, manganese, etc. Iron (Fe^{2+}) is a co-factor of enzyme catalase.

Properties of Enzymes

Proteinaceous nature
Enzymes are usually made up of proteins.

Enzyme specificity
Enzymes are highly specific in their action. Each enzyme act upon a specific substrate or a group of substrates.

Reversibility
- Almost all enzymatic reactions are reversible. However, reversibility is dependent upon energy requirements, availability of reactants, concentration of end products and pH.
- Enzymes are heat sensitive or thermolabile and operate between 20- 35°C (optimum temperature range). They become inactive at freezing temperatures and denature at higher temperature.

Catalytic property
A small quantity of enzymes can catalyse the transformation of a very large quantity of the substrate into an end product. Heat does not affect catalytic action of inorganic catalyst but inactivates the enzyme.

Nomenclature of enzymes

- According to international code of enzyme nomenclature, the name of each enzyme ends with an -ase and consists of double name. The first name indicates the nature of substrate upon which the enzyme acts and the second name indicates the reaction catalysed, *e.g.*, pyruvic decarboxylase catalyses the removal of CO_2 from the substrate pyruvic acid.

Classification of Enzymes

- **Oxidoreductases :** These are enzymes catalysing oxidation and reduction reactions by the transfer of hydrogen and/or oxygen. *E.g.*, alcohol dehydrogenase.
- **Transferases :** These enzymes catalyse the transfer of certain groups between two molecules. *E.g.*, glucokinase.
- **Hydrolases:** These are enzymes catalyse hydrolytic reactions. This class includes amylases, proteases, lipases, etc. *E.g.*, sucrase.
- **Lyases :** These enzymes are involved in elimination reactions resulting in the removal of a group of atoms from substrate molecule to leave a double bond. It includes aldolases, decarboxylases, and dehydratases. *E.g.*, fumarate hydratase.
- **Isomerases :** These enzymes catalyse structural rearrangements within a molecule. These are identified as racemases, epimerases, isomerases, mutases. *E.g.*, xylose isomerase.
- **Ligases or Synthetases :** These are the enzymes which catalyse the covalent linkage of the molecules utilising the energy obtained from hydrolysis of ATP, GTP. *E.g.*, glutathione synthetase, pyruvate carboxylase.

Mechanism of Enzyme Action

- The basic mechanism by which enzymes catalyse chemical reactions begins with the binding of the substrate to the active site on the enzyme. The active site is the specific region of the enzyme which combines with the substrate.
- The binding of the substrate to the enzyme causes changes in the distribution of electrons in the chemical bonds of the substrate and ultimately causes the reactions that lead to the formation of products. The products are released from the enzyme surface to regenerate the enzyme for another reaction cycle.

Two models to explain the mechanism of enzyme-substrate complex are as follows :

(i) **Lock and key model:** This hypothesis of enzyme action was put forward by **Emil Fischer** in 1894. According to this hypothesis, both enzyme and substrate molecules have specific geometrical shapes. In the region of active sites of surface, configuration of the enzyme is such as to allow the particular substrate molecules to be held over it, *i.e.*, a substrate molecule can be acted upon by a particular enzyme. This also **explains the specificity** of enzyme action.

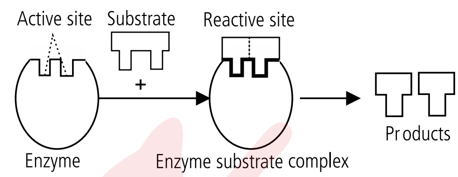


Fig.: Lock and key model

(ii) **Induced-fit model (Flexible model) :** This hypothesis was proposed by **Koshland** in 1959, which states that approach of a substrate induces a conformational change in the enzyme. It is more accepted model.

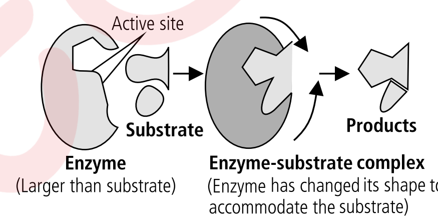


Fig.: Complex flexible model

Factors Affecting Enzyme Activity

- **Concentration of Substrate :** Increase in the substrate concentration gradually increases the velocity of enzyme activity within the limited range of substrate levels.
  A rectangular hyperbola is obtained when velocity is plotted against the substrate concentration. Three distinct phases (A, B and C) of the reaction are observed in the graph, where V = Measured velocity, V_{max} = Maximum velocity, S = Substrate concentration, K_m =Michaelis-Menten constant.

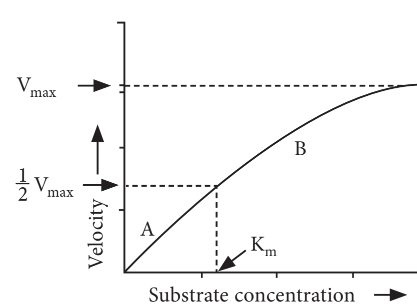


Fig.: Effect of substrate concentration on enzyme activity

- Michaelis-Menten equation mathematically illustrates the relationship between initial reaction velocity and substrate concentration. Here, the K_m (Michaelis Menten constant) is a mathematical derivative which indicates the substrate concentration at which the chemical reaction catalysed by an enzyme attains half its maximum velocity. K_m indicates affinity of the enzyme for its substrate. A high K_m indicates low affinity between enzyme and substrate, while a low K_m indicates strong affinity between them. For majority of enzymes, the K_m values are in the range of 10^{-5} to 10^{-2} moles.

- **Enzyme concentration :** The rate of an enzymatic reaction is directly proportional to the concentration of the substrate. The rate of reaction is also directly proportional to the square root of the concentration of enzymes, which implies; with increasing concentration of enzyme, rate of reaction increases.
- **Temperature :** The temperature at which an enzyme shows its highest activity is called **optimum temperature**. The optimum temperature for most of the enzymes is between 25°C-35°C. High temperature above 50° C destroys enzymes by causing their denaturation and very low temperature preserves the enzymes in the inactive state.
- **Effect of pH :** Enzymes work at their own optimum pH. A rise or fall in pH reduces enzymatic activity by changing the degree of ionisation of its side chains.
- **Other substances :** The enzymes action is also increased or decreased in the presence of some other substances such as co-enzymes, activators and inhibitors. Most of the enzymes are combination of a co-enzyme and an apoenzyme. Activators are the inorganic substances which increase the enzyme activity. Inhibitor is the substance which reduces the enzyme activity.

Concept of metabolism

- Metabolism is the sum of the chemical reactions that take place within each cell of a living organism and provide energy for vital processes and for synthesising new organic material.

Pathways of Metabolism

Catabolic pathways
It lead to formation of simpler structure from a complex biomolecule, *e.g.*, when we eat *chapati*, our gastrointestinal tract digests (hydrolyses) the starch to glucose units with help of enzymes and releases energy in form of ATP.

Anabolic pathways
It is called biosynthetic pathway that involves formation of a more complex biomolecules from a simpler structure, *e.g.*, synthesis of glycogen from glucose and protein from amino acids. These pathways consume energy.

- **Metabolic pool:** It refers to the reservoir of biomolecules in the cell on which enzymes can act to produce useful products as per the need of the cell. The concept of metabolic pool is significant in cell biology as it allows one type of molecule to change into another type *e.g.*, carbohydrates can be converted to fats and *vice-versa*.
- Catabolic chemical reaction of glycolysis and Krebs cycle only provide ATP but also makes available metabolic pool of biomolecules that can be utilized for synthesis of many important cellular components.
- The balance between catabolism and anabolism maintains homeostasis in the cell as well as in the whole body.

Secondary metabolites

Many plants, fungi and microbes of certain genera synthesise a number of small organic compounds which are not involved in primary metabolism (photosynthesis, respiration, protein and lipid metabolism) and seem to have no direct function in growth and development of organism. These compounds are called **secondary metabolites**. For example, gum, resin, rubber, etc.

Secondary metabolites

Terpenes	**Phenolics**	**Nitrogen-containing compounds**
They are made from mevalonic acid that is composed mainly of carbon and hydrogen.	They are made from simple sugars containing benzene rings, hydrogen and oxygen.	They contain nitrogen.

Economic importance of secondary metabolites :

1. Morphine was the first alkaloid isolated from plant *Papaver somniferum*. It is used as pain reliever and cough suppressant.
2. SMs like alkaloids nicotine and cocaine and the terpenes cannabinol are widely used for recreation and stimulation.
3. Characteristic flavours and aroma of cabbage and its relatives are caused by nitrogen and sulphur-containing chemicals, glucosinolates, protect these plants from many pests.
4. Tannins are added to wines and chocolate for improving astringency.
5. Since most of secondary metabolites are having antibiotic properties, they are also used as food preservatives.

Previous Years' Questions

1. In which of the following, RNA is double stranded?
(a) Reovirus (b) Wound tumour virus
(c) Bacteriophage phi ×174 (d) Both (a) and (b) *(2016)*

2. Steroids are characterised by ______ in their structure.
(a) carbon atom arranged in four interlocking rings
(b) magnesium atom arranged in the centre of tetrapyrrole
(c) two, six-carbon rings
(d) skeletal heterocyclic ring of hydrocarbons *(2018)*

3. Which one of the following carbohydrates is a heteropolysaccharide?
(a) Cellulose (b) Starch
(c) Glycogen (d) Hyaluronic acid *(2018)*

4. Match column-I with column-II considering the reference of enzymes and select the correct option.

Column-I	Column-II
(A) Transferase	I. Aldolase
(B) Hydrolase	II. Alcohol dehydrogenase
(C) Lyase	III. Sucrase
(D) Oxidoreductase	IV. Glucokinase

(a) A - III, B - I, C - II, D - IV
(b) A - IV, B - I, C - III, D - II
(c) A - II, B - III, C - I, D - IV
(d) A - IV, B - III, C - I, D - II *(2021)*

5. Protein digestive enzyme pepsin secreted in the duodenum of digestive system is most active at an optimum pH ________.
(a) 2 (b) 5.7
(c) 9.5 (d) 7.3 *(2021)*

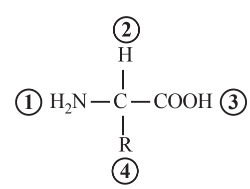
6. Given below are two statements.
Statement I : According to international code of enzyme nomenclature, first part in the name of an enzyme indicates the nature of substrate upon which the enzyme acts.
Statement II : The second part in the name of an enzyme indicates the reaction catalysed by it.
Considering above mentioned statements, choose the correct answer from the options given below.
(a) Statement I is correct but statement II is incorrect.
(b) Both statement I and statement II are correct.
(c) Both statement I and statement II are incorrect.
(d) Statement I is incorrect but statement II is correct. *(2021)*

7. Phenolics are made up of
(a) simple sugar containing benzene rings
(b) hydrogen
(c) oxygen
(d) all of these. *(2022)*

8. Which of the following statements about enzymes is incorrect ?
(a) Enzymes are denatured at high temperatures.
(b) Enzymes are mostly proteins but some are lipids also.
(c) Enzymes are highly specific.
(d) Enzymes require optimum pH and temperature for maximum activity. *(2023)*

9. Enzyme catalysts differ from inorganic catalysts in which of the following way(s)?
(a) Enzyme catalysts are smaller in size and lesser in weight in comparison to that of inorganic catalysts.
(b) Inorganic catalysts can work efficiently at high temperature but enzyme catalysts cannot (except few enzymes).
(c) Inorganic catalysts can work efficiently at high pressure but enzyme catalyst cannot.
(d) Both (b) and (c) *(2023)*

10. Which of the two groups of the given formula is involved in peptide bond formation between different amino acids?



(a) 2 and 3 (b) 1 and 3
(c) 1 and 4 (d) 2 and 4 *(2023)*

Hints & Explanations

1. (d) : Reovirus and wound tumour virus possess double stranded RNA. Bacteriophage ϕ × 174 and several bacterial viruses possess single stranded DNA.

2. (a) : Steroids consists of carbon atom arranged in four interlocking ring structures. Some common steroids are : cholesterol, vitamin D, steroid hormones and bile salts.

3. (d) : Cellulose, starch and glycogen are homopolysaccharides, while hyaluronic acid is a heteropolysaccharide.

4. (d)

5. (a) : Pepsin (in duodenum) is most active at an optimum pH of 2 (acidic).

6. (b) : According to international code of enzyme nomenclature, the name of each enzyme ends with an -ase and consists of double name. The first name indicates the nature of substrate upon which the enzyme acts and the second name indicates the reaction catalysed. For example, pyruvic decarboxylase catalyses the removal of CO_2 from the substrate pyruvic acid.

7. **(d)**

8. **(b) :** Almost all enzymes are functional proteins but lipids never act as enzyme.

9. **(b) :** Enzyme catalysts differ from inorganic catalysts in many ways, but one major difference is that inorganic catalysts work efficiently at high temperatures and high pressures, while enzymes get damaged at high temperatures (say above 40°C). However, enzymes isolated from organisms who normally live under extremely high temperature (*e.g.*, hot vents and sulphur springs), are stable and retain their catalytic power even at high temperatures (upto 80-90 °C). Thermal stability is thus, an important quality of such enzymes isolated from thermophilic organisms.

10. **(b) :** Proteins are polymers of different amino acids, which are linked by peptide (CONH) bond formed by dehydration (polymerisation) between COOH group of one amino acid and NH_2 group of next amino acid with the removal of H_2O.

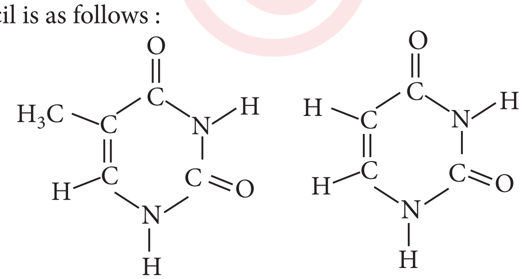
Solved Examples

Example 1 Which molecules form the basic components of carbohydrates? Give the general formula of carbohydrates.
Soln.: Carbohydrates are compounds made up of carbon, hydrogen and oxygen where hydrogen atoms and oxygen atoms generally occur in ratio of 2 : 1. Their general formula is $C_nH_{2n}O_n$.

Example 2 What do you understand by the antiparallel arrangement of DNA strands?
Soln.: The two DNA chains are antiparallel that is, they run parallel but in opposite directions. In one chain, the direction is $5' \rightarrow 3'$ while in the other it is $3' \rightarrow 5'$. The two chains are held together by hydrogen bonds between their bases.

Example 3 How is the bond between two amino acids formed? Name the bond formed.
Soln.: The bond between two amino acids is called peptide bond. Peptide bond is formed by the reaction between carboxyl (–COOH) group of one amino acid and amino (–NH_2) group of other amino acid with the elimination of water.

Example 4 Draw the structures of two pyrimidine bases.
Soln.: The structure of two pyrimidine bases thymine and uracil is as follows :



Thymine Uracil

Example 5 Explain anabolic and catabolic reactions with one example of each.
Soln.: Catabolic pathways lead to formation of simpler structure from a complex biomolecules, *e.g.*, when wheat, bread or *chapati* are eaten, the starch is hydrolysed by the gastrointestinal tract to glucose units with help of enzymes and releases energy in form of ATP (Adenosine triphosphate). Anabolic pathway is called biosynthetic pathway that involves formation of a more complex biomolecules from a simpler structure, *e.g.*, synthesis of glycogen from glucose and protein from amino acids. These pathways consume energy.

Example 6 How does temperature influence the enzyme action?
Soln.: An enzyme is active within a narrow range of temperature. The temperature at which an enzyme shows its highest activity is called optimum temperature. Enzymatic activity decline both below and above the optimum temperatures. Enzyme becomes inactive below optimum temperature. Low temperature preserves the enzyme in the inactive state. High temperature destroys enzymes by causing their denaturation. This occur at above 50°C.

Example 7 Mention three major classes of RNA present in cell along with their functions.
Soln.: The three major types of RNA are:
(i) Messenger RNA (*m*RNA) – It brings instructions from the DNA for the formation of particular type of polypeptide.
(ii) Ribosomal RNA (*r*RNA) – In combination with proteins, it forms the small and large subunits of ribosomes.
(iii) Transfer RNA (*t*RNA) – It is involved in the transferring amino acids to ribosomes for polypeptide synthesis.

Example 8 Explain the difference between a nucleotide and a nucleoside with an examples.
Soln.: Differences between nucleoside and nucleotide are as follows:

	Nucleoside	Nucleotide
(i)	Nucleoside is a compound formed by the union of a nitrogenous base with a pentose sugar.	Nucleotide is a compound formed by the union of a nitrogenous base, a pentose sugar and phosphate.
(ii)	Two examples of nucleoside are adenosine and uridine.	Two examples of nucleotide are adenylic acid and thymidylic acid.

Example 9 What is Michaelis-Menten constant?

Soln.: Michaelis-Menten constant (K_m) is defined as the substrate concentration at which the chemical reaction catalysed by an enzyme attains half its maximum velocity. Its value generally lies between 10^{-5} to 10^{-2} moles.

Example 10 What is diosgenin? Write its function.

Soln.: In plants, sterols exist chiefly as phytosterols. Yam Plant (*Dioscorea*) produces a steroid compound called diosgenin. It is used in the manufacture of antifertility pills, *i.e.* , birth control pills.

Daily Practice Problems

Introduction

1. Match the following columns and select the correct option.

	Column-I		Column-II
(A)	Amino acids	(1)	Lipids
(B)	Glycerol	(2)	Proteins
(C)	Nucleotides	(3)	Polysaccharides
(D)	Sugars	(4)	Nucleic acids

	A	B	C	D
(a)	2	1	4	3
(b)	1	2	3	4
(c)	3	2	1	4
(d)	4	3	2	1

2. Relation between amino acid and protein is similar to the one found between
(a) glucose and fructose
(b) thymine and uracil
(c) nucleosides and nucleic acid
(d) nucleotides and nucleic acid.

Biomolecules in the cell

3. The storage molecule in plants is _____ whereas in animals is _______.
(a) glycogen, starch
(b) inulin, starch
(c) starch, glycogen
(d) starch, cellulose

4. Starch is a homopolymer of
(a) glucose (b) sucrose
(c) galactose (d) fructose.

5. Amylose is a ________ polymer and amylopectin is a ________ polymer respectively.
(a) branched, unbranched
(b) unbranched, branched
(c) unbranched, unbranched
(d) branched, branched.

6. Which of the following sugars cannot be hydrolysed further to yield simple sugars?
(a) Ribose (b) Maltose
(c) Sucrose (d) Lactose

7. Which one of the following natural polymers is found in insect?
(a) Pectin (b) Chitin
(c) Cellulose (d) Glycogen

8. Select the statement which is correct for polysaccharides.
(a) These are macromolecules formed by polymerisation of monosaccharide units.
(b) These are carbohydrates categorised as class of micromolecules.
(c) These are short chains of sugars attached with phosphate groups.
(d) These are heteropolymers of glucose only as in cellulose.

9. Which one is a disaccharide?
(a) Glucose (b) Ribose
(c) Maltose (d) Glycogen

10. Which among the following is homopolysaccharide?
(a) Glucose (b) Ribose
(c) Sucrose (d) Cellulose

11. Match column I with column II and choose the correct option.

	Column I		Column II
A.	Chitin	I.	Polymer of β-glucose
B.	Glycogen	II.	Exoskeleton of cockroach
C.	Starch	III.	Liver and muscles
D.	Cellulose	IV.	Polymer of α-glucose

(a) A – I; B – IV; C – III; D – II
(b) A – II; B – III; C – IV; D – I
(c) A – IV; B – I; C – III; D – II
(d) A – I; B – IV; C – II; D – III

12. Lipids are insoluble in water because lipid molecules are
(a) hydrophilic (b) hydrophobic
(c) neutral (d) zwitter ions.

13. Match the items in column I with items in column II and choose the correct option.

	Column-I		**Column-II**
(A)	Sterol	(1)	Diosgenin
(B)	Compound lipid	(2)	Wax
(C)	Simple lipid	(3)	Phospholipids

	A	**B**	**C**
(a)	1	3	2
(b)	2	3	1
(c)	3	2	1
(d)	3	1	2

14. A typical fat molecule is made up of
(a) one glycerol and one fatty acid molecule
(b) three glycerol and three fatty acid molecules
(c) three glycerol molecules and one fatty acid molecule
(d) one glycerol and three fatty acid molecules.

15. A steroid compound produced by *Dioscorea* is
(a) glucosinolates (b) tannins
(c) diosgenin (d) morphine.

16. Read the given statements and select the correct option.
Statement 1 : Saturated acids are without double bonds.
Statement 2 : There are one or more double bonds between carbon atoms in unsaturated fatty acids.
(a) Both statements 1 and 2 are true.
(b) Both statements 1 and 2 are false.
(c) Statement 1 is true but statement 2 is false.
(d) Statement 1 is false but statement 2 is true.

17. A saturated fatty acid is
(a) palmitic acid (b) oleic acid
(c) linoleic acid (d) both (b) and (c).

18. Select the correct statement(s) regarding lipids.
(a) Lipids are generally water insoluble.
(b) Lipids could be simple fatty acids.
(c) Some lipids have phosphate group in them.
(d) All of these

19. The tertiary structure of the proteins is held by
(a) hydrogen bonds
(b) disulphide bonds
(c) Van der Waal's force
(d) both (a) and (b).

20. If all the peptide bonds of a protein are broken, then the remaining part is
(a) amide (b) oligosaccharide
(c) polypeptide (d) amino acid.

21. Match column-I with column-II and select the correct option.

	Column-I		**Column-II**
(A)	Haemoglobin	(1)	Basic amino acid
(B)	Lysine	(2)	Quaternary structure
(C)	Myoglobin	(3)	Secondary structure
(D)	β-pleated sheet	(4)	Tertiary structure

	A	**B**	**C**	**D**
(a)	4	1	3	2
(b)	3	4	1	2
(c)	1	3	2	4
(d)	2	1	4	3

22. The alpha helix and beta pleated sheets are the examples of which level of protein organisation?
(a) Primary structure
(b) Secondary structure
(c) Tertiary structure
(d) Quaternary structure

23. Which of the following is basic amino acid?
(a) Alanine (b) Arginine
(c) Valine (d) Serine

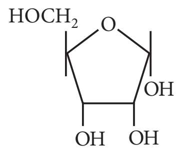
24. Read the given statements and select the correct option.
Statement 1 : Prosthetic groups are bound loosely to the apoenzyme.
Statement 2 : Prosthetic groups are non-protein constituents that makes the enzyme catalytically active.
(a) Both statements 1 and 2 are correct.
(b) Statement 1 is correct but statement 2 is incorrect.
(c) Statement 1 is incorrect but statement 2 is correct.
(d) Both statements 1 and 2 are incorrect.

25. Which of the following statements is correct about the proteins?
(a) The sequence of amino acids in a protein represents the secondary structure.
(b) The helices of proteins are always left handed.
(c) Functional proteins have 2-D conformation.
(d) Proteins are heteropolymers of amino acids.

26. In a protein, amino acids are linked by
(a) peptide bonds (b) glycosidic bonds
(c) hydrogen bonds (d) none of these.

27. In a double stranded DNA, ratio of adenine and thymine/ guanine and cytosine is equal. It was first given by
(a) Franklin and Wilkins (b) F. Meischer
(c) Watson and Crick (d) Erwin Chargaff.

28. Given below is the structural formula of



(a) sucrose (b) ribose
(c) glucose (d) deoxyribose.

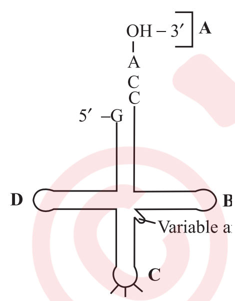
29. The most abundant RNA in cell is
(a) *m*RNA (b) *t*RNA
(c) *r*RNA (d) catalytic RNA.

30. Match column I with column II and choose the correct option.

	Column I		Column II
(A)	Purine	(i)	DNA
(B)	Nucleic acid	(ii)	Uracil
(C)	Pyrimidine	(iii)	Adenine

	A	B	C
(a)	(ii)	(iii)	(i)
(b)	(i)	(ii)	(iii)
(c)	(iii)	(i)	(ii)
(d)	(i)	(iii)	(ii)

31. Identify the labels A, B, C and D in the given structure of *t*RNA and select the correct option.



	A	B	C	D
(a)	Anticodon loop	TΨC loop	Amino acid binding site	DHU loop
(b)	Amino acid binding site	TΨC loop	Anticodon loop	DHU loop
(c)	Amino acid binding site	DHU loop	Anticodon loop	TΨC loop
(d)	Amino acid binding site	DHU loop	TΨC loop	Anticodon loop

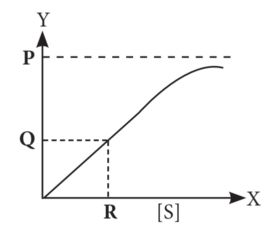
32. Nucleosides are made up of
(a) purine and pyrimidines only
(b) purine/pyrimidine and sugar
(c) purine/pyrimidine, sugar and phosphate
(d) ribose sugar and phosphate.

33. How many nitrogenous bases are common in DNA and RNA nucleotides?
(a) Two (b) Three
(c) Four (d) Only one

34. Double hydrogen bonds occur in DNA between
(a) adenine and thymine
(b) uracil and thymine
(c) adenine and guanine
(d) thymine and cytosine.

35. The K_m value of the enzyme is the value of the substrate concentration at which the reaction reaches zero to
(a) zero (b) 2 V_{max}
(c) 1/2 V_{max} (d) 1/4 V_{max}.

36. The given graph depicts the change in concentration of the substrate on enzyme activity. Identify P, Q and R and select the correct option.



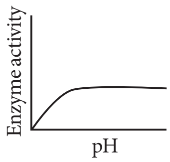
	P	Q	R
(a)	V_{max}	$\frac{V_{max}}{2}$	K_m
(b)	K_m	V_{max}	$\frac{V_{max}}{2}$
(c)	$\frac{V_{max}}{2}$	K_m	V
(d)	$\frac{V_{max}}{2}$	K_m	V_{max}

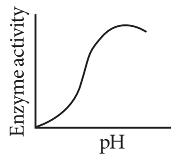
37. The proteinaceous molecule that joins a non-proteinaceous prosthetic group to form a functional enzyme is called
(a) cofactor (b) apoenzyme
(c) holoenzyme (d) isoenzyme.

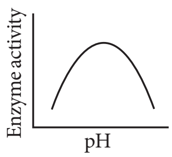
38. Glucokinase that catalyses conversion of Glucose to Glucose-6-P belongs to the category
(a) transferases (b) lyases
(c) oxidoreductases (d) isomerases.

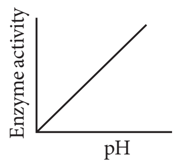
39. Read the given statements and select the correct option.
Statement 1 : Enzymes are like inorganic catalysts and influence the speed of biochemical reactions.
Statement 2 : Enzymes have high catalytic power.
(a) Both statement 1 and 2 are true.
(b) Both statements 1 and 2 are false.
(c) Statement 1 is true but statement 2 is false.
(d) Statement 1 is false but statement 2 is true.

40. Choose the correct graph showing the effect of change in pH on the enzyme activity.

(a) 

(b) 

(c) 

(d) 

41. Which of the following statements is incorrect regarding enzymatic activity?
(a) It increases with increase in substrate concentration upto the saturation point.
(b) It is highest at optimum pH value.
(c) It initially decreases with increase in pH value.
(d) It initially increases with increase in temperature and then decreases.

42. Which enzyme catalyses the hydrolysis of sucrose?
(a) Lyases (b) Transferases
(c) Ligases (d) Hydrolases

43. Catalytic cycle of enzyme can be described by the following steps.
1. Substrate binds to the active site of the enzyme.
2. Active site of the enzyme breaks the chemical bonds of the substrate forming a new enzyme–product complex.
3. Shape of the enzyme is altered due to binding of the substrate.
4. Product of the reaction is released.

What is the correct sequence of enzyme action?
(a) $1 \rightarrow 2 \rightarrow 3 \rightarrow 4$ (b) $1 \rightarrow 3 \rightarrow 2 \rightarrow 4$
(c) $2 \rightarrow 1 \rightarrow 3 \rightarrow 4$ (d) $2 \rightarrow 3 \rightarrow 1 \rightarrow 4$

44. Tannin is a secondary metabolite which is synthesised *via*
(a) mevalonic acid pathway
(b) phenylpropanoid pathway
(c) glycolytic pathway
(d) C_3 pathway.

45. Glucosinolates is a nitrogen and sulphur containing compound which can be used
(a) as a pesticide
(b) as a food preservative
(c) in recreation and stimulation
(d) in improving astringency of chocolates.

46. Identify whether the given reactions are anabolic or catabolic.
I. Maltose → Glucose + Glucose
II. Amino acids → Proteins
(a) I-Catabolic; II-Catabolic (b) I-Anabolic; II-Catabolic
(c) I-Catabolic; II-Anabolic (d) I-Anabolic; II-Anabolic

47. Anabolic pathways ____(i)____ energy and catabolic pathways __(ii)__ energy. Identify (i) and (ii) and select the correct option.
(a) (i)-consume, (ii)-release
(b) (i)-release, (ii)-consume
(c) (i)-consume, (ii)-consume
(d) (i)-release, (ii)-release

48. All the chemical reactions occurring in living organisms are called
(a) metabolism (b) anabolism
(c) catabolism (d) enzymatic.

49. Nicotine is a/an
(a) alkaloid (b) stimulant
(c) depressant (d) both (a) and (b).

50. Which of the following properties of secondary metabolites allows it to be used as a food preservative?
(a) It is used as stimulant.
(b) It improves food astringency.
(c) It improves our food preference.
(d) It has antibiotic properties.

ANSWER KEYS

1. (a)	**2.** (d)	**3.** (c)	**4.** (a)	**5.** (b)	**6.** (a)	**7.** (b)	**8.** (a)	**9.** (c)	**10.** (d)
11. (b)	**12.** (b)	**13.** (a)	**14.** (d)	**15.** (c)	**16.** (a)	**17.** (a)	**18.** (d)	**19.** (d)	**20.** (d)
21. (d)	**22.** (b)	**23.** (b)	**24.** (c)	**25.** (d)	**26.** (a)	**27.** (d)	**28.** (b)	**29.** (c)	**30.** (c)
31. (b)	**32.** (b)	**33.** (b)	**34.** (a)	**35.** (c)	**36.** (a)	**37.** (b)	**38.** (a)	**39.** (a)	**40.** (c)
41. (c)	**42.** (d)	**43.** (b)	**44.** (b)	**45.** (a)	**46.** (c)	**47.** (a)	**48.** (a)	**49.** (d)	**50.** (d)